Math 615A

Derived Categories in Algebraic Geometry

Fall 2017 Course Information

Time: TuTh 14:00-15:30

Place: TBA.

Instructor: Kai Behrend

Office hours: TBA
in Math Annex 1213.

Text Book: **D. Huybrechts.** Fourier-Mukai Transforms in Algebraic Geometry.

Prerequisites: A full year of graduate algebraic geometry, such as Math 532/533 (equivalent to the first 3 chapters of Hartshorne).

Course Outline: This is an introduction to the derived category and its applications in algebraic geometry. Since the introduction of derived categories into mathematics by Grothendieck and his school in the early 1960s, they were mostly considered a formal tool for dealing with such things as derived functors, hypercohomolgoy, and spectral sequences. More recently, it was discovered, by Mukai and the Russian school, that derived categories are, in fact, interesting as geometric objects. In some regards, they are more basic than the varieties giving rise to them.

After going over the basic definitions of triangulated categories and derived categories, we will be interested mostly in the derived category of coherent sheaves on a projective algebraic variety. The first highlight is Grothendieck-Verdier duality, a generalization of Serre duality. Next comes the result of Bondal and Orlov, which shows that for varieties with ample canonical or anti-canonical bundle, the derived category determines the variety it comes from. Thus, derived categories of varieties with trivial canonical bundle are of particular interest.

Then we study Fourier-Mukai transforms between derived categories. These are derived versions of correspondences. We study exceptional objects and auto-equivalences. We will

then round out the course with selections from the remaining chapters of the book.

Marking: There will be occasional homework assignments, which your mark will be based on.