MATH 515
Partial Differential Equations of Fluid Mechanics

January - April, 2018

Lecture summary and references: pdf file (this file is also updated in the owncloud folder)

Goal

This course introduces the mathematical theory for the partial differential equations (PDE) modeling the inviscid and viscous incompressible fluids, namely, the incompressible Euler equations and incompressible Navier-Stokes equations. A graduate student in either engineering, mathematics, or physics can hope to learn an important branch of the PDE theory motivated by fluid mechanics.

Topics

Here is the tentative outline. It can be adjusted according to audience background and feedback.

1. Mathematical background
    a. Lebesgue integral and L^p spaces
    b. weak derivative and Sobolev spaces
    c. weak convergence
    d. solutions for the heat equation in a domain, Galerkin and semigroup methods

2. An introduction to incompressible fluid flows
    a. derivation of the Euler and Navier-Stokes equations
    b. symmetry groups and conserved quantities; some exact solutions
    c. Leray's formulation and Hodge/Helmholtz decomposition

3. Incompressible Euler equations
    a. The vorticity-stream formulation
    b. solution by energy method
    c. The particle-trajectory method

4. Incompressible Navier-Stokes equations
    a. weak solutions, existence
    b. strong solutions, uniqueness and regularity
    c. mild solutions

References

We will mostly cover selected sections from the following.

1. Vorticity and Incompressible Flow, by Majda and Bertozzi.
2. Lectures on Navier-Stokes equations, by Tsai

Files of these books and other references will be available in a public owncloud folder, whose link will be given.

Prerequisites

The required mathematical background will be covered in the first part of the course. Most of them are covered in MATH 516, which is encouraged. However we will not assume MATH 516, in order to broaden the audience base.

Evaluation

The evaluation is based on homework assignments and class participation.

Instructor and lectures

Instructor: Dr. Tai-Peng Tsai, Math building room 109, phone 604-822-2591, ttsai at math.ubc.ca.

Lectures: We will discuss the lecture time in the first meeting (10am, Wednesday January 3, at IKB Learning Centre room 157). Please email me if you cannot attend it although you are interested.

Office hours: By appointment (Tsai's schedule: TBA).

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