MATH 305:201, Applied Complex Analysis

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Basic information

- Course website:
  `http://www.math.ubc.ca/~sbach/teaching/2017W/305/Info.html`
- Time and room: MWF 12-13 in TBA
- My office: MATH 228
- Contact me: sbach@math.ubc.ca
- Office hours: Mondays-Tuesdays 8:30-10:00, or by appointment
- Textbooks:
  Fundamentals of Complex Analysis with Applications to Engineering and Science, E.B. Saff and A.D. Snider, Pearson (third edition)
  Complex Analysis, volume II of Princeton Lectures in Analysis, E.M. Stein and R. Shakarchi, Princeton University Press

About the course

Complex analysis is the study of complex valued functions defined on the complex plane. It is simultaneously a beautiful mathematical theory having ramifications far beyond the theory of functions, and an extremely useful toolbox for applications in physics, engineering, and generally, applied mathematics.

This course aims at a non pedantic and enlightening exposition of the central mathematical results, most of which can be traced back to Cauchy's theorem: if a function is complex differentiable in a disk, then its integral along any closed curve inside the disk vanishes. The applications that will be discussed along the way run from methods to compute the sum of series or integrals of real-valued functions, to the dynamics of oscillatory systems or of incompressible fluids, to heat transport and wave propagation. Fourier series and the Fourier transform, whose importance for applications cannot be overstated, will also be discussed.

Outline of the material

1. Preliminaries
   - Complex numbers
   - Polar form, powers and roots
   - Sets in the complex plane
2. Functions
   - Continuity
   - Differentiability and the Cauchy-Riemann equations
   - Holomorphic functions
   - Power series
   - The exponential and logarithm functions
3. Integration
   - Contours
   - Integration along contours
   - Primitives
4. Cauchy's theorem and applications
   - Cauchy's theorem
   - Toy contours and evaluation of integrals
   - Cauchy's integral formulas
   - Liouville's theorem
5. Meromorphic functions
   - Zeros and poles
   - Laurent series
   - The residue formula
   - Evaluation of integrals
   - The argument function and the winding number
6. The Fourier transform
   - Fourier and inverse Fourier transforms
   - Elementary properties
   - Application to differential equations

Homework, Exams and Final Grade

On submitted work

All assertions require a proof unless the problem states otherwise. No matter the operative word ('find', 'solve', 'establish', 'calculate', 'determine',...), you must justify your answer.

Written work should be presented carefully, in complete English sentences, and with sufficient detail. A correct sequence of formulas will only merit partial credit, an unstructured cloud of formulas will merit none.

Homework assignments

Starting the second week, there will be an exercise sheet every week. Each problem set will be due at the beginning of class on the day indicated.

Learning mathematics from lectures or a textbook only is hopeless: it is absolutely essential for your understanding to work with new concepts and try to solve problems directly related to the course material. Independently of the points towards the final grade you may receive on your homework solutions, it is crucial to work on the problem sets in order to understand the material and to do well in the exams.

Although you are encouraged to discuss the problems with your peers, each of you must submit an independent written solution. Do not mix sharing ideas with sharing submitted work.

Exams

There will be two midterm exams in class lasting 50 minutes each,

on January 31st and March 7th.

There will be a final exam in the usual exam period.

Final Grade

The final grade is computed as such:

Problem sets: 20%; Midterm exams: 15% each; Final: 50%.

In calculating your score for the problem sets, I will drop your two lowest scores. These include missed assignment.

If you are to miss a midterm, let me know in advance of your legitimate reason. In case of an emergency, please contact me when the emergency is over and provide some proof of it. In both cases, your absence will be noted and the missed work will not count towards the final grade. Otherwise, the missed work will receive the grade 0. A student must finish a significant amount of term work in order to pass.

In the case of the final exam, the students should contact the Department of Mathematics office and the missed final will be handled in a formal way.