

## MATH 223: LINEAR ALGEBRA

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### Basic information

- Course website:  
<http://www.math.ubc.ca/~sbach/teaching/2017W/223/Info.html>
- Time and room: MWF 10-11 in MATX 1100
- My office: MATH 228
- Contact me: [sbach@math.ubc.ca](mailto:sbach@math.ubc.ca)
- Office hours:  
Monday 11:00-12:00, Tuesday 13:30-14:30, Friday 11:00-12:00 or by appointment
- Textbook: *Linear algebra*, S. Friedberg, A. Insel and L. Spence, Pearson (fourth edition)

### About the course

This course explores the concept of linearity, both at a theoretical level and a computational level. Linear algebra is an essential tool not only for algebra itself, but for analysis (numerics, functional analysis, differential equations), for computer science (search engines, computer graphics, data analysis) or quantum mechanics (as emphasized in Heisenberg's point of view). The course will concentrate on mathematical aspects and present complete proofs of all results.

Beyond the concrete subject at hand, linear algebra is an excellent corner of mathematics for a first serious contact with higher mathematics. The emphasis is neither on solving puzzles, nor on playing with numbers, nor on working out computations, but on understanding fundamental concepts (e.g. linearity), structures (e.g. a vector space), and operations within them (e.g. a linear transformation). One of the interests of this course will be for students to learn further what is a rigorous mathematical proof and how to construct one. It will require and foster the capacity for abstract thinking, while rooting this abstraction in concrete examples.

Needless to say, this course is aimed at excellent students.

## Outline of the material

1. Vector spaces
  - Definition
  - Subspaces
  - Linear (in)dependence
  - Bases and dimension
2. Linear transformations and matrices
  - Definition of a linear transformation
  - Kernel and range
  - Matrix representations
  - Composition and matrix multiplication
  - Invertible transformations and matrices, isomorphisms
3. Systems of linear equations
  - Elementary operations
  - The rank of a matrix
  - General solution of a linear system
4. Determinants
  - Introduction and properties
  - Construction of the determinant
  - Invertibility
  - Existence and uniqueness
  - Formulas
5. Introduction to spectral theory
  - Eigenvalues and eigenvectors
  - Diagonalizability
  - Real and complex eigenvalues
  - The Cayley-Hamilton theorem
6. Inner product spaces
  - Inner products
  - Orthogonality
  - Symmetric and hermitian matrices
  - The spectral theorem

## Homework, Exams and Final Grade

### On submitted work

All assertions require a proof unless the problem states otherwise. No matter the operative word ('find', 'solve', 'establish', 'calculate', 'determine',...), you must justify your answer.

Written work should be presented carefully, in complete English sentences, and with sufficient detail. A correct sequence of formulas will only merit partial credit, an unstructured cloud of formulas will merit none.

### Homework assignments

Starting the second week, there will be an exercise sheet every week. Each problem set will be due at the beginning of class on the day shown.

Learning mathematics from lectures or a textbook only is hopeless: it is absolutely essential for your understanding to work with new concepts and try to solve problems directly related to the course material. Independently of the points towards the final grade you may receive on your homework solutions, it is crucial to work on the problem sets in order to understand the material and to do well in the exams.

Although you are encouraged to discuss the problems with your peers, each of you must submit an independent written solution. Do not mix sharing ideas with sharing submitted work.

### Exams

There will be two midterm exams in class lasting 50 minutes each,  
on October 4th and November 8th.

There will be a final exam in the usual exam period.

### Final Grade

The final grade is computed as such

Problem sets: 20%; Midterm exams: 15% each; Final: 50%.

In calculating your score for the problem sets, I will drop your two lowest scores. These include missed assignment.

If you are to miss a midterm, let me know in advance of your legitimate reason. In case of an emergency, please contact me when the emergency is over and provide some proof of it. In both cases, your absence will be noted and the missed work will not count towards the final grade. Otherwise, the missed work will receive the grade 0. A student must finish a significant amount of term work in order to pass.

In the case of the final exam, the students should contact the Faculty of Science office and the missed final will be handled in a formal way.