Course Outline

MATH 566: Theory of Optimal Transportation

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Course Material and Topics:

The aim of this course is to provide an overview of the theory of optimal transportation, one of the most active research directions in analysis, partial differential equations, geometry, and probability. This theory is based on the following variational problem, first initiated by Monge in 19's century: What is the most efficient way of moving a mass distribution to another, while one needs to pay cost for the transportation? (For example, consider the problem of matching water resources to residential areas.) This simple looking problem is a starting point of a lot of mathematical developments in optimization, geometry, nonlinear partial differential equations, probability, with applications to many other disciplines including economics, computer science, biology, and meteorology. In this course, a focus will be on its connections to the theory of partial differential equations and geometric/functional inequalities. If time permits, the course will cover a few recent progress in the field, where the choice of topics will be discussed as the course goes along.

- Basic existence/uniqueness and characterization of optimal transportation: Kantorovich duality. Cyclical monotonicity. Geometry of optimal transportation
- Brenier's polar factorization theorem
- A PDE aspect of the theory: Monge-Ampère equations
- Geometry on the space of probability measures: displacement convexity
- Geometric/functional inequalities
- Evolution equations as gradient flows

List of Further Topics

- Multi-marginal optimal transport and Wasserstein barycenters.
- Optimal martingale transport.
- Regularity of optimal transportation.
- Constrained optimal transport and BV estimates.
- L^1 optimal transport and the needle decomposition.
- Optimal transport and geometry: Ricci curvature and Ricci flow.
- Applications to image processing, data analysis, and machine learning.
- Applications to meteorology: semi-geostropic equations
- Applications to economics.

Learning Objectives:

- Students will be prepared to read recent research articles in optimal transportation and related fields.

- Students will be able to prove several important geometric/functional inequalities and acquire alternative perspectives on measure theory and partial differential equations, different from those usually taught in first year graduate courses.
- Students will be able to apply and integrate basic concepts in measure theory, convex analysis and geometry, to develop new approaches to studying variational problems and partial differential equations.

Prerequisites:

First year graduate level measure theory is desirable and some knowledge of partial differential equations will be helpful.

Evaluation:

The instructional format for the course will consist of lectures of 3 hours per week.

The mark will be determined on two homework assignments. Additional coursework such as oral presentation is a possibility.

Rationale for Course:

The basic ideas in optimal transportation theory are rather simple, but their applications are powerful and they sometimes give surprising connections between things that are not seemingly related. This is why more researchers in science in general are becoming interested in the subject, and it will be beneficial for students both in pure and applied mathematics to get used to some of the essential ideas in optimal transportation as their general backgrounds; especially those who are pursuing research in analysis, geometry, probability, and PDE, as well as those in scientific computing, optimization, mathematical modelling, mathematical biology, image and data processing, to name a few. The course material is suitable for first or second year graduate students (even for advanced undergraduate students with solid background in measure theory) and prepares them to do their own research in the subject and related fields.

Textbooks and References:

Optional Textbooks:

- Cedric Villani: Topics in Optimal Transportation, AMS 2003.
- Filippo Santambrogio: Optimal Transport for Applied Mathematicians. Birkhauser 2015. Available at https://www.math.u-psud.fr/~filippo/OTAM-cvgmt.pdf

Other useful references:

- Cedirc Villani: Optimal Transport: old and new. Springer 2009. Available at http://math.univ-lyon1.fr/homes-www/villani/Cedrif/B07D.StFlour.pdf
- Nestor Guillen and Robert McCann: Five lectures on optimal transportation: geometry, regularity and applications. Available at http://www.math.toronto.edu/mccann/publications

- Filippo Santambrogio: {Euclidean, Metric, and Wasserstein} Gradient Flows: an overview. Available at http://cvgmt.sns.it/media/doc/paper/3165/surveyGradFlows.pdf
- Luigi Ambrosio and Nicola Gigli: A User's Guide to Optimal Transport Available at http://cvgmt.sns.it/media/doc/paper/195/users_guide-final.pdf