THE UNIVERSITY OF BRITISH COLUMBIA

Greg Martin

Professor
Department of Mathematics

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Teaching

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Lectures: Mondays, Wednesdays, and Fridays, 10–11 AM, room MATH 203 ([Mathematics building](#))
Office hours: Tuesdays 2–3 PM and Fridays 11 AM–12:30 PM, or by appointment
Office: room MATH 212 ([Mathematics Building](#))
Email address: gerg@math.ubc.ca

Course description: The first few weeks will be spent quickly covering the foundations of elementary number theory: divisibility, congruences, prime numbers, and so on, some of which might already be familiar to you. Emphasis will be on a level of mastery sufficient for you to teach the material. Once we have this foundation, we will move on to roots of polynomial congruences, arithmetic and multiplicative functions, binary quadratic forms, and parametrizing Pythagorean triples. The two most important topics of the course are primitive roots and quadratic reciprocity. Topics that might also be covered include simple Diophantine equations, Diophantine approximation, and continued fractions.

Course textbook: *An Introduction to the Theory of Numbers*, by Niven, Zuckerman, and Montgomery, 5th edition (required). We will cover roughly the following sections:

- Sections 1.2–1.3: divisibility, gcds, primes
- Sections 2.1–2.3 and 2.6–2.8: congruences, solutions of congruences, Chinese remainder theorem, prime power modulus, prime modulus, primitive roots and power residues
- Sections 3.1–3.6: quadratic residues, quadratic reciprocity, Jacobi symbol, sums of two and four squares
  - (if time permits: binary quadratic forms)
- Sections 4.2–4.4: arithmetic functions, Möbius inversion, recurrence functions
  - (if time permits: A.4, linear recurrences)
- Sections 5.1 and 5.3: solving $ax + by = c$ in integers, Pythagorean triangles
  - (if time permits: 5.2 and 5.4, simultaneous linear equations and assorted examples)
- Sections 6.1–6.3: Farey sequences, rational approximations, irrational numbers (as time permits)
- Sections 7.1–7.8: the Euclidean algorithm, uniqueness, infinite continued fractions, (best possible) approximations of irrationals by rationals, periodic continued fractions, Pell's equation (as time permits)

Notes to undergraduates: For all practical purposes, *MATH 437 is an honours course*! It treats roughly the same material as MATH 312 and 313 combined, and will take nearly twice as much work as either of those classes. Note that a student cannot have credit for both MATH 312 and MATH 437, nor for both MATH 313 and MATH 437. To enroll in MATH 437, an undergraduate student must have already taken, or be taking simultaneously, one of MATH 320 or MATH 322.

The word "elementary" in the title does not mean the course isn't difficult; rather it means that the course doesn't use techniques from real or complex analysis or from abstract algebra. The course will not require any particular background in number theory. What is required is "mathematical sophistication", which certainly includes being able to understand and write proofs. Be forewarned that this course will be taught at the level of a graduate course. Honours students typically will be well-equipped to succeed in this course.

Evaluation: The course mark will be based on (approximately) weekly homework assignments (65% of the final mark; the lowest homework score will automatically be ignored), engaged participation in in-class group problem sessions (10% of the final mark), and one final exam (25% of the final mark) whose date and location are still to be determined. The final exam will be a typical closed-book exam, with problems very much like the homework problems (indeed, there will probably be significant inclusion of actual homework/discussion

problems on the final exam). In the case of extreme disparity between homework/discussion marks and exam marks, the instructor may use his discretion in assigning a final course mark.

Your homework will be marked on correctness, completeness, rigor, and elegance. A correct answer will not earn full marks unless it is completely justified, in a rigorous manner, and written in a logical sequence that is easy to follow and confirm. I plan on being pedantic about completeness of solutions (for example, if you invoke Euler's theorem to assert that $a^{\varphi(q)} \equiv 1$ (mod q), you need to explicity acknowledge the fact that a must be relatively prime to q). Part of the goal of this course is to provide training and practice at writing proofs with sufficient rigor to be accepted by research journals.

You are very welcome to come by my office hours and ask questions about the lecture material, homework/discussion problems, clarity of style in proof writing, or related mathematical content. If office hours conflict with your schedule, you may make an appointment with me via email; you are also welcome to just drop by my office and see if I'm there—about half the time, I'll be free to talk right then. Students are allowed to consult one another concerning the homework problems, but *your submitted solutions must be written by you in your own words.* If two students submit virtually identical answers to a question, both can be found guilty of plagiarism.

Other than the group work, *no handouts will be distributed in class*. All homework assignments and any other course materials will be posted on this course web page below. **Homework solutions must be prepared in LaTeX** and submitted to me in PDF format via email; please add your name to the filename before submitting your homework (for example, GregMartin-homework1.pdf). I will supply LaTeX templates with each assignment. All homeworks are due before the beginning of class (9:59 AM) on the indicated days.

- Homework #0: due Monday, September 12. Download both the TeX file and the PDF file.
- Homework #1: due Monday, September 19
- Homework #2: due Monday, September 26
- Homework #3: due Monday, October 3
- Homework #4: due Friday, October 14
- Homework #5: due Friday, October 21
- Homework #6: due Friday, October 28
- Homework #7: due Friday, November 4
- Homework #8: due Friday, November 18
- Homework #9: due Friday, November 25
- Homework #10: due Friday, December 2

- Group Work #1: Wednesday, September 14
- Group Work #2: Wednesday, September 21
- Group Work #3: Wednesday, September 28
- Group Work #4: Wednesday, October 5
- Group Work #5: Monday, October 17
- Group Work #6: Monday, October 24
- Group Work #7: Monday, October 31
- Group Work #8: Monday, November 7
- Group Work #9: Monday, November 21
- Group Work #10: Monday, November 28