

## Math 423/502: Topics in Algebra: Introduction to Commutative algebra and Representation Theory

Instructor: Julia Gordon

Office: Math 217

e-mail: gor at math.ubc.ca

**Text:** There is no required text, but the following books will be useful:

- Atiyah and Macdonald “Introduction to commutative algebra”
- D.S. Dummit, R.M. Foote, “ Abstract Algebra”.
- “A *Singular* introduction to commutative algebra” by Greuel and Pfister (available online the library)
- J.-P. Serre, “ Linear Representations of Finite groups”.
- S. Bosch, “Algebraic geometry and Commutative Algebra” (available online

Links to some online notes will also be posted.

**Course outline:** Approximately two thirds of this intense “Required background in Algebra” course will be focused on Commutative Algebra, and for the last third, we will switch to Representation Theory of finite groups. The goal of the Commutative Algebra part is to serve as preparation for a first course in Algebraic Geometry. It will cover the following topics:

- Quick reminder of rings and ideals; prime ideals and maximal ideals. Main example: polynomial rings.
- Hilbert basis theorem and Gröbner bases.
- Nilpotent elements. Nilradical and Jacobson radical.
- Local rings and localization.
- Modules: tensor product, extension and restriction of scalars.
- Noetherian and Artinian rings.
- Primary decomposition in Noetherian rings.
- Krull dimension.
- Hilbert’s Nullstellensatz.
- Time permitting, we will discuss topics related to smoothness and singularities.

The Representation Theory part of the course will provide a quick introduction to representation theory of finite groups, which is an important prerequisite for many subjects, including Lie Theory, Algebraic Number Theory, and even some aspects of Harmonic Analysis. The topics will include:

- The notion of a group representation
- Characters
- Representations as modules over the group ring
- Induced representations
- Pontryagin dual of an Abelian group

**Background expectations.** The official prerequisites are Math 412 or Math 501; the real pre-requisite not listed in the calendar is Math 323 (or at least some familiarity with rings and modules). You will need good understanding of linear algebra.

**Marking.** The mark will be based on homework (60%) and a take-home final essay (40%). Interaction and collaboration on homework is encouraged, but if you collaborate, please acknowledge this in writing. Collaboration on the midterm and final is not permitted.