# Math 419/545 <br> Stochastic Processes/Probability II Winter 2017 

Instructor: Ed Perkins, Math Annex 1207
Contact: perkins@math.ubc.ca
Lectures: MWF 10:00-10:50 in Math 202.
Course webpage: http://www.math.ubc.ca/~perkins/teaching.html
Office hours: M 11-12:30; F 1:10-2:40
Text: R. Durrett. Probability: Theory and Examples. Version 4.1 available for free download at the author's webpage at https://services.math.duke.edu/~rtd/PTE/pte.html
If you have any trouble downloading it, a copy will eventually be on the course webpage.

Course outline: This course is a continuation of Math 418/544. Together they give a comprehensive introduction to measure theoretic probability which should be ideal for those wishing to study probability, or use it as a tool in analysis, statistics, mathematical biology, economics, finance or applied mathematics. The course will focus on stochastic processes, including the study of martingales or fair games, Markov Chains, and continuous time stochastic processes including Brownian motion. This corresponds to Chapters 5-8 and A. 3 of Durrett's text.

## 1. Conditional Expectation

Part I. Brief Overview of topics covered in 418/544
Part II. Connections with conditional densities and elementary conditioning.
2. Martingale Theory.

Gaussian bounds (Azuma/Freedman/Bennett/Hoeffding inequality)
Predictable processes and discrete stochastic integral $(H \cdot X)_{n}$
Upcrossing lemma and Martingale convergence theorem
Applications to branching processes and harmonic functions.
Uniformly integrable martingales, convergence in $L^{1}$, maximal inequalities,

Reverse martingales, examples,
Optional stopping
Applications to Radon-Nikodym theorem, strong law of large numbers.
Further reading: Probability with Martingales by D. Williams.
3. Kolmogorov Extension Theorem
4. Markov Chains

Strong Markov property, recurrence and transience, stationary measures, the convergence theorem, coupling, random walk.

Further reading: Markov Chains by James Norris

## 5. Brownian Motion

Construction, Markov and martingale properites, strong Markov property, path properties, functional central limit theorem (Donsker's theorem), applications to random walk.

Further reading:
Dynamical Theories of Brownian Motion by E. Nelson.
Introduction to stochastic integration (1st or 2nd ed.) by K.L. Chung and R. Williams.

Convergence of Probability Measures by P. Billingsley.
6. Ergodic Theorems

Birkhoff's ergodic theorem, subadditive ergodic theorem, applications.
Further reading: Probability by L. Breiman.

Further Reading. Here are a few textbooks that cover much of the course material:

Probability
R. Ash. Real Analysis and Probability
O. Kallenberg. Foundations of Modern Probability.
A. Klenke. Probability Theory: A Comprehensive Course.

Measure Theory
R. Ash. Real Analysis and Probability (or Real Analysis)
H. Royden Real Analysis
W. Rudin Real and Complex Analysis

## Evaluation:

Homework will be assigned regularly (5 or 6 in total) for $2 / 3$ of the grade. Late submissions will not be accepted.

There will be a 2 hour midterm 5:30-7:30, Thurs. March 23 for $1 / 3$ of the grade.

