UNIVERSITY OF BRITISH COLUMBIA

Math 606D Jan-Apr 2016

Topics in Differential Equations: Existence and bifurcation of Navier-Stokes equations

This course studies the existence and bifurcation of several boundary value problems for the Navier-Stokes equations. Two recent breakthroughs in the mathematical fluid mechanics are the existence theorem of the boundary value problem in all bounded 2D and axisymmetric 3D domains of Korobkov-Pileckas-Russo, and the existence theorem of large forward self-similar solutions of Jia-Sverak. We will present these results, starting with the necessary background, and show one instance of their connection. A common task of these topics is to obtain a priori bounds of the solutions. Topics of study include Euler equations, Sard's theorem, Leray-Schauder degree and local Leray solutions. Most problems treated are either time-independent or time-periodic.

Prerequisite: MATH 516. Other relevant materials will be reviewed during the course.

Topics:

1. Boundary value problem of stationery Navier-Stokes equations
   (a) existence with zero boundary condition
   (b) small data uniqueness, nonuniqueness
   (c) existence when boundary is connected
   (d) bifurcation of Navier-Stokes coupled with heat convection
   (e) bifurcation of Couette-Taylor flows
2. Korobkov-Pileckas-Russo approach for 2D Boundary value problem
   (a) Sard's theorem for Sobolev functions
   (b) Bernoulli Law for stationary Euler equations in Sobolev spaces
   (c) geometry of level sets of stream functions of 2D Euler equations in Sobolev spaces
   (d) 2D existence for general boundary
3. Existence of large forward self-similar solutions
   (a) Local Leray solutions in whole space and a priori bounds
   (b) Existence of self-similar solutions by Leray-Schauder degree
   (c) Non-uniqueness conjecture
   (d) Existence in half space using KPR approach
   (e) Discretely self-similar solutions, strong and weak

References: I will supply the references during the term. The following are main references.

1. Galdi, G. P.: An introduction to the mathematical theory of the Navier-Stokes equations, 2nd ed., Springer, 2011.

2. Jia, H, Šverák, V.: Local-in-space estimates near initial time for weak solutions of the Navier-Stokes equations and forward self-similar solutions, *Invent. Math.* 196 (2014), no. 1, 233–265.

3. Korobkov, M.V.; Pileckas K.; Russo, R.: Solution of Leray's problem for stationary Navier-Stokes equations in plane and axially symmetric spatial domains, *Ann. of Math.* 181 (2015), no. 2, 769–807.

4. Temam, R.: Navier-Stokes equations : theory and numerical analysis, 1973 1st ed., 1977 2nd ed., or 1984 3rd ed..

5. Tsai, T.-P., Lectures on Navier-Stokes equations, 2014.

Evaluation: The course evaluation will be based on presentation. I will make a list of papers for you to choose from, and provide you the electronic files.

Instructor: Tai-Peng Tsai, Math 109, phone 604-822-2591, ttsai@math.ubc.ca.