

# Introduction to Harmonic Analysis - Math 541

Spring 2016

- **Instructor:** *Malabika Pramanik*
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  - **Office hours:** *To be announced.*
- **Web page:** The course website is  
<http://www.math.ubc.ca/~malabika/teaching/ubc/spring16/math541/index.html>
- Homework assignments and all relevant course information (such as changes to office hours if any, or solutions to homework problems if needed) will be posted here.
- **Text:** There are no required textbooks. The following textbooks are recommended.
- *Lectures on Harmonic Analysis (2003)*, by *T.H. Wolff*, AMS, ISBN: 978-0-8218-3449-7.
  - *An Introduction to Harmonic Analysis (3rd edition)*, by *Y. Katznelson*, Cambridge, ISBN: 978-0-521-54359-2.
  - *Singular Integrals and Differentiability Properties of Functions (1970)*, by *E. Stein*, Princeton University Press, ISBN: 0-691-08079-8.
  - *Introduction to Fourier Analysis on Euclidean Spaces (1971)*, by *E. Stein*, and *G. Weiss*, Princeton University Press, ISBN: 0-691-08078-X.
  - *Harmonic Analysis: Real-variable Methods, Orthogonality and Oscillatory Integrals (1993)*, by *E. Stein*, Princeton University Press, ISBN: 0-691-03216-5.
  - *Classical and Modern Fourier Analysis*, by *L. Grafakos*.
- **Course outline :** The cope topics of the course are the following:
1. *Basic material concerning Fourier series, Fourier transform and Fourier inversion*
    - Fourier basis for  $L^2(\mathbb{T})$
    - Convolution
    - Approximate identities
    - Temperate distributions
    - Some applications
  2. *Convergence of Fourier series*
    - Decay of Fourier coefficients
    - Uniform convergence of Fourier series

- Pointwise convergence and almost everywhere divergence
- Norm convergence

### 3. *Interpolation of operators*

- Complex methods (Riesz-Thörin theorem, analytic interpolation)
- Real methods (Marcinkiewicz interpolation theorem)
- Applications (Hausdorff-Young inequality, Young's convolution inequality, fractional integration, Hardy-Littlewood maximal theorem).

### 4. *Singular integral operators*

- Calderón-Zygmund kernels
- Some multiplier operators
- The Calderón-Zygmund decomposition
- $L^p$  boundedness of Calderón-Zygmund singular integral operators
- Homogeneous distributions, Hilbert transform, Riesz transform.

### 5. *Littlewood-Paley theory*

- Almost orthogonality in Hilbert spaces, Cotlar-Knapp-Stein lemma
- A square function that characterizes  $L^p$
- Variations and applications

Time permitting, we will also consider other special topics.

- **Lectures :** Monday, Wednesday, Friday 10 am - 11 am in Mathematics Annex 1118.
- **Grading Policy :** To be announced after the first week.