

Math 419/545-Stochastic Process/Probability II

Schedule: MWF 10-11 in Math 104

Instructor: Ed Perkins, Math Annex 1207, perkins@math.ubc.ca, 604-822-6670

Text: Probability: Theory and Examples, 4th ed. by R. Durrett, Duxbury Press.

Prerequisite: Math 418/544 or consent of the instructor.

Description: This course is a continuation of Math 418/544. Together they give a comprehensive introduction to measure theoretic probability which should be ideal for those wishing to study probability, or use it as a tool in analysis, statistics, finance or applied mathematics. The course will continue the study of martingales or fair games and give introductions to the study of Markov Chains, ergodic theory and continuous time stochastic processes including Brownian motion. This corresponds to Chapters 5–8 of Durrett's text.

Outline

1. Martingales

Part I. Brief Overview of topics covered in 418/544

Gaussian bounds (Azuma/Freedman/Bennett/Hoeffding inequality)

Predictable processes and discrete stochastic integral $(H \cdot X)_n$

Upcrossing lemma and Martingale convergence theorem

Applications to branching processes and harmonic functions.

Part II. Further topics

Uniformly integrable martingales, convergence in L^1 , maximal inequalities,

Reverse martingales, examples,

Optional stopping

Applications to Radon-Nikodym theorem, strong law of large numbers.

Further reading: Probability with Martingales by D. Williams.

2. Markov Chains

Strong Markov property, recurrence and transience, stationary measures, the convergence theorem, coupling.

Further reading: Markov Chains by James Norris

3. Ergodic Theorems

Birkhoff's ergodic theorem, subadditive ergodic theorem, applications.

Further reading: Probability by L. Breiman.

4. Brownian Motion

Construction, Markov and martingale properties, strong Markov property, path properties, functional central limit theorem (Donsker's theorem), applications to random walk.

Further reading:

Dynamical Theories of Brownian Motion by E. Nelson.

Introduction to stochastic integration (1st or 2nd ed.) by K.L. Chung and R. Williams.

Convergence of Probability Measures by P. Billingsley.

5. Stochastic Processes

Topics to be determined depending on time and interest.

Grading

Homeworks (60%); Final Exam (40%)