

UBC Mathematics 402(201)

MWF 11:00-11:50, room MATH 204

<http://www.math.ubc.ca/~loew/m402/>

Overview: This course deals with the theory of minimization problems in which the choice variable is a function of one real variable. The classical theory as developed from Bernoulli through Euler to the present will be developed with careful attention to both theoretical precision and practical consequences.

Readings: Lecture notes will appear regularly on the course web page, [/www.math.ubc.ca/~loew/m402/](http://www.math.ubc.ca/~loew/m402/).

Prerequisite: C+ or better in either Math 320 or Math 301, or consent of the instructor.

Grading: 50% for the term (weekly homework, one test), 50% for the final exam.

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| CLASS TEST DATE: WEDNESDAY 25 FEBRUARY 2015. |
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Outline:

- I. The basic problem
 - A. Formulation of the problem; examples.
 - B. Descent directions and abstract necessary conditions.
 - C. The Euler-Lagrange Equation.
 - D. Regularity of extremals.
 - E. Invariance of extremals.
 - F. Strong local minima and the Weierstrass condition.
- II. Modifications of the basic problem.
 - A. Free endpoint problems.
 - B. Problems with several dependent variables.
 - C. Problems with several independent variables (sketch).
 - D. Isoperimetric problems.
 - E. The Weierstrass and Legendre conditions.
- III. Convexity.
 - A. Convex functions in finite-dimensional optimization.
 - B. Convex Lagrangians and sufficiency.
- IV. Second-order necessary conditions.
 - A. The accessory problem.
 - B. The Legendre condition.
 - C. The Jacobi condition.
 - D. Envelopes of extremals.
- V. Sufficient conditions.
 - A. Locally convex problems.
 - B. Equivalent problems.
 - C. The classical necessary conditions are almost sufficient.
 - D. Fields of extremals.
 - E. Existence and regularity theory.
- VI. Dynamic Programming.
 - A. Verification Functions.
 - B. The Principle of Optimality.
 - C. The Value Function.
 - D. The Hamilton-Jacobi Equation.
- VII. Classical Mechanics.
 - A. Hamilton's principle of least action.
 - B. Canonical transformations (optional).
 - C. Hamilton-Jacobi theory.

Topics V to VII may be truncated if time runs out. Student requests for other related topics are welcome.

