

Ordinary Differential Equations

Special Section for Mech 2

Overview. This is a special section of MATH 255 dedicated to the MECH 2 program. This award-winning collaborative model for Engineering education involves a cohort of students who enjoy an intensive, integrated experience in which lectures, labs, and tutorials are closely coordinated. Details of the Mech 2 program are online at

<http://mech.ubc.ca/undergraduate-students/mech-2/>

Instructor. Professor Philip D. Loewen, office MATH 207, loew@math.ubc.ca. See also

<http://math.ubc.ca/~loew/mech2/>

Learning Activities. Professor Loewen will present 28 formal lectures and lead occasional scheduled Question-and-Answer (QA) sessions. In addition, there will be weekly small-group tutorials led by a Teaching Assistant. Weekly homework assignments and computer labs, offered in collaboration with the companion course MECH 221 (*Engineering Science I*), will also support student learning for many of the core concepts in the course.

Lecture Outline (Tentative).

1. Particular Solutions: Particular solutions of $L[y] = f$, when f is either polynomial or exponential.
2. Exponential Shift, I: Reducing $L[y] = g$ with g in family PE to a new problem $L'[y] = f$, with polynomial f . Real exponentials only.
3. Complementary Solutions: Relevance of $L[y] = 0$; cataloguing all solutions.
4. Initial-value Problems: A 3-step plan for $L[y] = f$: Guess one solution; find all the rest; select if needed. Uniqueness from exp shift with rate a .
5. Complex Numbers: Arithmetic, esp division. Argand plane. Euler's formula. Pics of $\Re(e^{st})$ for various complex s , map in complex plane. Expressing real PET fcn as real part of complex PE fcn.
6. Exponential Shift, II: Solving $L[y] = g$ with g in family PET and complex exponentials.
7. Physics and stability: Mass-Spring Systems; Electrical Analogies; Over/Under/Critical Damping; Decay.
8. Transfer Functions: Mother Nature's Beer Fridge and related problems
9. Resonance and Beats: Resonance, Near Resonance and Beats – mechanical and electrical implications. OR Transfer Function analysis.
10. First-order linear scalar equations: Integrating factors for $y' + p(t)y = f(t)$, mixing applications [slightly off the main track, a nice diversion]
11. Linear ODE systems: First Order Linear ODE Systems; Phase Space; Eigenvalue connections
12. Phase Plane – eigenvals real: Linear Systems: Phase Plane Portraits (Real Eigenvalues)
13. Phase Plane – eivs complex: Linear Systems (cont): Complex Eigenvalue Case

14. Matrix Exponentials: Beautiful unifying view of scalar and vector situations
15. ODE systems with forcing: Linear Systems (cont): Inhomogeneous Systems, esp resonance
16. Laplace Transforms (LT): Laplace Transforms (LT): introduction
17. LT Properties: LT Properties and calculations; Shift Theorem #1
18. LT solves ODE's: LT: solving DE's & transfer function
19. SISO Frequency Response: LT: Steady-state response to sinusoidal input
20. Discontinuous Inputs: LT: Heaviside unit step; Shift Theorem #2
21. Impulsive Inputs: LT: Convolution. Impulses if time permits.
22. Autonomous Nonlinear ODE's: Autonomous Equations (cont): Critical Points, Phase Line, Linearization, Stability
23. Approximating Integrals: Approximating definite integrals (Averages, Trapezoidal Rule, Simpson's Rule)
24. Robocat; Numerical Solutions: Autonomous ODE's with 2D unknowns as motivation;
25. Taylor Polynomials: Taylor Polynomials, with error analysis
26. Approximating Derivatives: Numerical differentiation, with O -notation for errors
27. Numerical Solution of DE's: Numerical Solution of Differential Equations – Euler and Improved Euler
28. Matlab ODE45: Numerical Solution of Differential Equations – Runge-Kutta and ODE45

Textbooks (Required). Both resources below are free online.

[JL] Lebl, Jiri, Notes on Diffy Qs: Differential Equations for Engineers.
<http://www.jirka.org/diffyqs/>

[WFT] Trench, William, Elementary Differential Equations with Boundary Value Problems. <http://digitalcommons.trinity.edu/mono/9/>

Evaluation. Students' grades will be determined by their demonstrated understanding on weekly homework assignments, weekly submissions in labs (both physical labs and computer labs), weekly quizzes, and two final examinations. MECH 2 is so deeply interdisciplinary that almost all of these activities will combine mathematical work with engineering analysis from a related subject. Therefore each student's grade in this section of MATH 255 will be the same as that student's grade in the 12-credit companion course MECH 221. Much more detail is provided in the *Mech 2 Handbook* and on the UBC Connect site dedicated to the program.