# MATHEMATICS 401 2012/13 T2 Green's Functions and Variational Methods

Instructor: George Bluman
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Office hours: by appointment (or drop-in if available)

**Optional Texts:**
Haberman, Applied Partial Differential Equations (Prentice-Hall)
Gelfand & Fomin, Calculus of Variations (Dover)

**References:**
Courant & Hilbert, Methods of Mathematical Physics, Volumes I & II (Interscience)
Duff & Naylor, Differential Equations of Applied Mathematics (Wiley)
Carrier & Pearson, Partial Differential Equations (Academic Press)
Copson, Partial Differential Equations (Cambridge)
Gustafson, Introduction to Partial Differential Equations and Hilbert Space Methods (Wiley)
Tikhonov & Samarskii, Equations of Mathematical Physics (Pergamon)
Zauderer, Partial Differential Equations of Applied Mathematics (Wiley-Interscience)
Goldstein, Classical Mechanics (Addison-Wesley)
Landau & Lifshitz, Mechanics (Addison-Wesley)
Bliss, Calculus of Variations (MAA monograph)
Weinstock, Calculus of Variations (Dover)
B & Kumei, Symmetries and Differential Equations (Springer)
B, Cheviakov & Anco, Applications of Symmetry Methods to Partial Differential Equations (Springer)

**Topics:**

I. Green's functions for linear ODEs & PDEs — 6 weeks

distributions (Dirac delta function, other distributions)
Green's functions for ODEs
Green's functions for PDEs

--homogeneous heat equation in one space dimension—infinite, semi-infinite and finite domains, fundamental solution, sources and sinks
--inhomogeneous heat equation in one space dimension
--adjoint operator
--Green's function for a Dirichlet problem
--Dirichlet problem for Laplace's equation

- Laplace's equation in two space dimensions—source solution, Dirichlet problems for a disc, for a semicircular region, for a half-plane, for an arbitrary domain (conformal mapping)
- Laplace's equation in three space dimensions—source solution

--Neumann problem for Laplace's equation
--linear elliptic equations (Dirichlet, Neumann and Robin problems)

- how to find formally Green's function for an arbitrary domain in terms of eigenvalues and eigenfunctions

--linear parabolic equations in *n* space dimensions for an arbitrary domain

--linear wave equations in *n* space dimensions for an arbitrary domain

- domain of dependence, range of influence, characteristic cone

II Calculus of variations 3.5 weeks

connections with classical mechanics—Lagrangian, path of least action

extremizing a functional

--normed linear space, admissible functions, first variation, necessary condition, Gâteaux variation, Euler derivative, Euler equation, fixed endpoints, free endpoints, brachistochrone problem

extremal problems with side conditions (constraints)

. --extremizing functions with function constraints; Lagrange multipliers

--extremizing functionals with functional and/or function constraints

- isoperimetric problem
- geodesics on a sphere

III Eigenfunction expansions 2.5 weeks

IV Rayleigh-Ritz and finite element methods 1 week

**Pre-requisites:** Math 316 or 257; Math 217 or 227 or 317; Math 400 is helpful but not necessary
**Grading:** 50% from weekly homework assignments (based on the best 10 assignments); 50% from the final exam. There will be no midterms in this course.
**N.B. A student must pass the final examination in order to pass this course.**

There will be 10-12 homework assignments (each will be due at the beginning of a class on the due date). Late homework will not be accepted. *Copying solutions from another student, from the web or any other source and turning them in as your own is a violation of the Academic Code and will lead to severe punishment.*

Not all homework problems will be graded each week. Solutions will be sent to you by e-mail.