

**THE UNIVERSITY OF BRITISH COLUMBIA**

Mathematics 414 Section 201

**No calculators allowed**

**Final exam begins at 3:30 pm and ends at 6:00 pm**

FINAL EXAM

December 10, 2013

NAME

STUDENT NUMBER

[10] 1. If  $\sec x - \tan x = 3$ , then what is the numerical value of  $\csc x$ ?  
(Sarah's problem)

[10] 2. What is the time between 5pm and 6pm when the hour hand and the minute hand of a clock make exactly 90 degrees? (Taeyun's problem)

[10] 3. A right triangle with one base length 7 has a circle with radius 3 inscribed in it. What is the area of the triangle? (Shaun's problem)

[10] 4. Assume there are 500 grams of orange juice made from 92% water and 8% concentrate? We divide this juice into two glasses. Glass A has 350 grams and glass B has 150 grams. Suppose we let the contents of glass A evaporate (water only) until we have 300 grams of mass. We also add 50 grams of water to glass B. We want to have an even concentration between A and B. How much orange concentrate must be added to B to match A's concentration? (Tae Won's problem)

[10] 5. A princess visits an island inhabited by two tribes. Members of one tribe always tell the truth, and members of the other tribe always lie.

The princess comes to a fork in the road. She needs to know which road leads to the castle so as to avoid the fire-breathing dragon and rescue the prince from the wizard holding him captive in the castle. (Although the princess doesn't know it, the south road leads to the castle and the north road leads to the dragon.)

Standing at this fork in the road is a member of each tribe, but the princess can't tell which tribe each belongs to. What question should she ask to find the road to the castle? Explain why your question leads to the desired result.

6.

[2](a) What does it mean to say that 2 sets A and B have the same cardinal number?

[4](b) Prove that the set E of non-negative even integers has the same cardinal number as the set N of all non-negative integers.  $N = \{0, 1, 2, 3, \dots, n, \dots\}$ .

[4](c) Show that if A is a set of 3 letters ( $A = \{a, b, c\}$ ), then  $\text{card } A + \text{card } N = \text{card } N$ .

[10] 7. Physics majors and math majors attend a conference. While there they take a small IQ test out of 40. The math majors' average IQ score was 35 whereas the physics majors' average IQ score was 25. When the scores are combined, the average score on the IQ test was 31. What was the ratio of physics majors to math majors at the conference?

8. A tub has a flat bottom with area 5000 square cm and vertical sides. There is water in the tub to the depth of 20 cm.

[5] (a) A concrete cube with sides 25cm is placed on the floor of the tub. How much does the water level rise?

[5] (b) A second identical cube is placed on the floor of the tub. How much further does the water level rise now?

[10] 9. Bernadette is playing a game at Hogwarts. There are 10 devil's snares in front of her in a line. Assume that every time she casts a lumos (i.e. a spell) on a live snare, it dies, and every time she casts on a dead snare it comes back to life.

Starting from the first snare, in the first round of the game she casts lumos on every second snare. In the second round she casts on every third snare, and in the third round she casts on every fourth snare. In the fourth round she casts on every second snare. Assume that every snare that enters a round dead will be revived as a result of a spell during that round and remain alive at the beginning of the next round OR if not revived during that round will in any case be revived at the beginning of the next round. Assume also that every snare that enters 2 successive rounds alive will either die as the result of a lumos during the second round and enter the next round dead OR will die at the beginning of the next round. Assume that the lumos are cast after the beginning of each round. How many snares are alive after the 4th round, assuming all the snares started out alive?

10. In a certain country the following coins are in circulation: 1 cent, 2 cents, 5 cents, 10 cents, 20 cents, 50 cents and 1 dollar.

[5](a) You can pay 77 cents with 4 coins. Show that you can pay 4 dollars with 77 coins.

[5](b) Suppose you can pay  $A$  cents with  $B$  coins. Prove that you can pay  $B$  dollars with  $A$  coins.