

Final Examination Duration: 150 minutes*This test has 12 questions on 15 pages, for a total of 120 points.*

First Name: _____ Signature: _____

Family Name: _____ Date: _____

- Read all the questions carefully before starting to work.
- You should show your work. Partial credit is possible for each problem.
- Correct answers without the accompanying work will not receive full credit.
- You may leave you answers in **calculator ready** form unless otherwise stated.
- Continue on the back of the previous page if you run out of space and **leave a note**.
- You are permitted one (double-sided) formula sheet.
- **Electronic devices of any kind (including calculators, cell phones, etc.) are NOT permitted.**

Question:	1	2	3	4	5	6	7	8	9	10	11	12	Total
Points:	10	10	10	10	10	10	10	10	10	10	10	10	120
Score:													

Student Conduct during Examinations

1. Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
2. Examination candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
3. No examination candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no examination candidate shall be permitted to enter the examination room once the examination has begun.
4. Examination candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
5. Examination candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
 - (i) speaking or communicating with other examination candidates, unless otherwise authorized;
 - (ii) purposely exposing written papers to the view of other examination candidates or imaging devices;
 - (iii) purposely viewing the written papers of other examination candidates;
 - (iv) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
 - (v) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
6. Examination candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
7. Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
8. Examination candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

10 marks

1. a) A piece of gum and a chocolate bar cost one dollar and ten cents in total. The bar costs a dollar more than the gum. How much does the gum cost?

Answer:

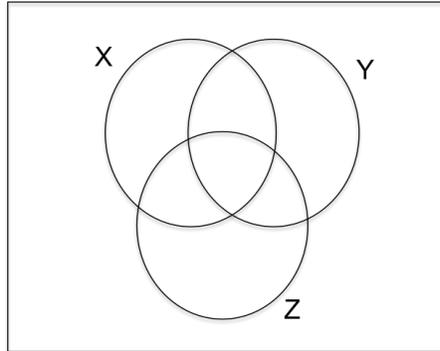
- b) Suppose that $\frac{4}{7}$ of a gallon of gas filled $\frac{3}{11}$ of a tractor's gas tank. How many gallons would it take to fill the tractor's gas tank completely?

Answer:

10 marks

2. a) Use the following Venn diagram to shade in the part that corresponds to:

$$(X \cap Y) \cup (Y^c \cap Z)$$



- b) A veterinarian surveys 50 of his patrons. He discovers that 32 have dogs, 27 have cats, and 12 have fish. Twelve have dogs and cats, 7 have cats and fish, and 4 have all three kinds of pets. How many patrons have a dog or a fish, but not both?

Answer:

10 marks

3. Explain why the Diophantine equation

$$54x + 81y = 43 \tag{1}$$

is impossible to solve, but

$$54x + 43y = 81 \tag{2}$$

can be solved.

10 marks

4. a) Find the prime factorization of 1105.

Answer:

b) Find the lcm of 120 and 156.

Answer:

c) Is $2^8 \times 3^5 \times 7^4$ divisible by 11 without a remainder? Justify your answer.

Answer:

10 marks

5. Answer each of the following numbers without doing actual division. Provide a brief explanation for full marks.

a) Is 321655252 divisible by 6?

Answer:

b) Is 1872514746 divisible by 8?

Answer:

c) Is 517425354 divisible by 9?

Answer:

d) Is 787509173 divisible by 11?

Answer:

10 marks

6. Drake was trying to impress Rihanna with his maths ability by estimating $\sqrt{69}$, which he claimed was “eight something”. He didn’t have much luck after that though. Find the next digit (so, 8.?) by using one iteration of the Babylonian method and then converting the result to a decimal number.

Recall to approximate \sqrt{N} , we start out with some guess X_0 (preferably the floor value of \sqrt{N}). Then we compute a new estimate for \sqrt{N} as follows:

$$X_{new} = \frac{1}{2} \left(X_{Old} + \frac{N}{X_{Old}} \right).$$

Answer:

10 marks

7. The math club sells two types of shirts: traditional nerdy and super-duper nerdy. The traditional shirts sell for \$28 each and the super-duper ones sell for \$31 each. If the total revenue from Monday was is \$1000, how many of the super-duper nerdy shirts did they sell?

Answer:

10 marks

8. a) Explain the Law of Large Numbers in your own words.

b) A drawer contains 10 black socks, 6 white socks, and 4 red socks.

i) How many socks must be pulled out to guarantee a matching pair?

Answer:

ii) How many socks must be pulled to guarantee a pair of white socks?

Answer:

iii) If you randomly pulled out 2 socks, what is probability that they would be a matching pair? State your answer as a reduced fraction.

Answer:

10 marks

9. Lie detector tests can often clear you as a suspect, but cannot be used in court to prove guilt. Let's look one of the major reasons why. Let's say that about 95% of all people are honest. Suppose that a lie detector test correctly classifies 90% the liars and 90% of the honest people. That is, when a person has lied the test has success rate of 90% to classify the person as a liar. When the person told the truth, the test has a 90% success rate to classify them as honest. Now suppose that a random person is in for an interview and failed the lie detector test; that is, the machine concluded that the person was lying! What is the probability that they actual lied? State your answer as a percentage.

Answer:

10 marks 10. a) Evaluate $44 \times 35 + 88$ in \mathbb{Z}_{16}

Answer:

b) Evaluate $72^6 - 43 \times 22$ in \mathbb{Z}_{23}

Answer:

c) Evaluate $6 \div 7$ in \mathbb{Z}_{31}

Answer:

10 marks

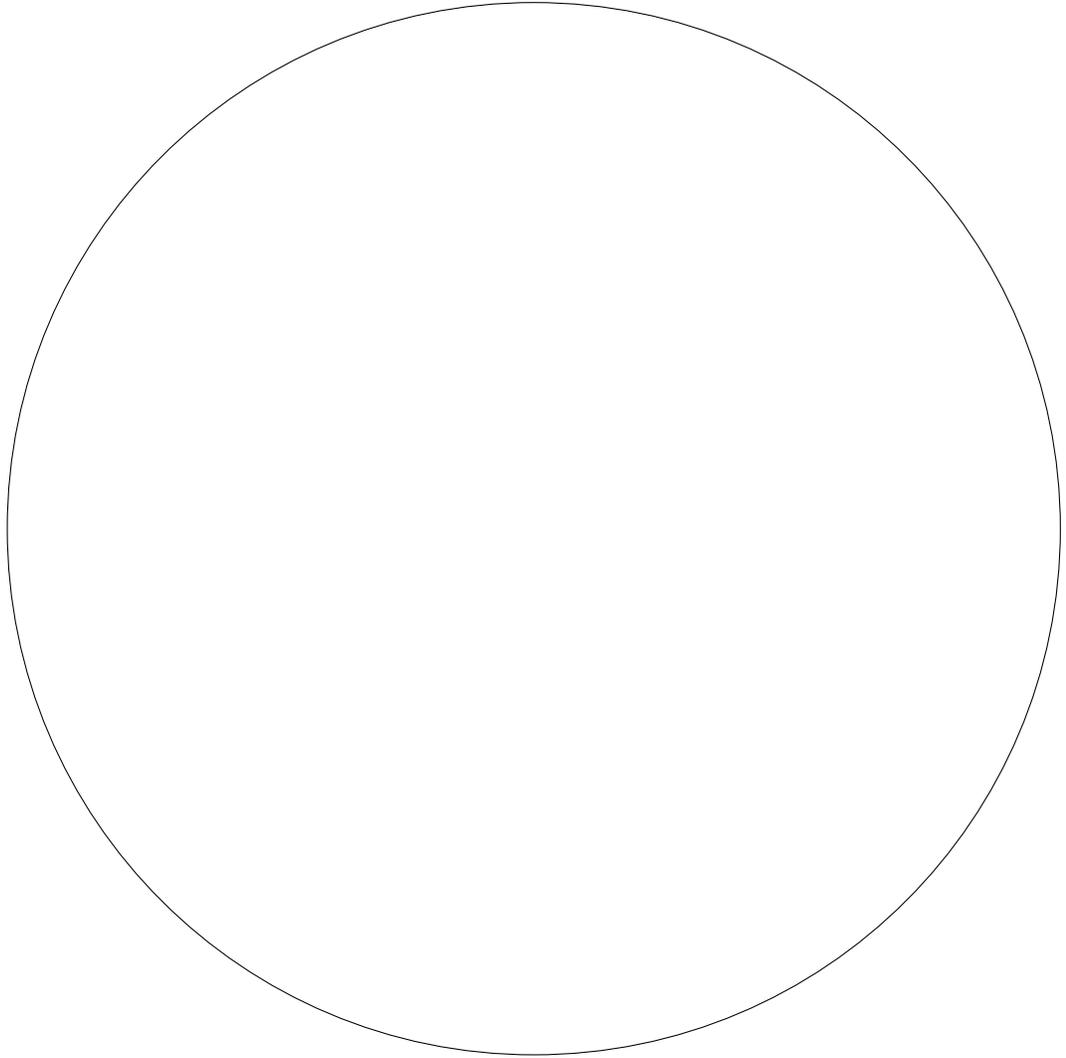
11. Determine the missing digit, Y, of following ISBN:

978-0-6Y4-06664-9

Answer:

10 marks

12. a) Use your compass and straightedge to find the exact center of the circle below.



12. (con't)

- b) The Golden ratio occurs in nature all over the place. Its exact value is $\frac{1+\sqrt{5}}{2}$. Given that the line segment below is 1 unit long. Use your compass and straightedge to draw a line segment whose length is exactly the Golden Ratio. Please provide a brief list of steps to outline your “plan” for partial credit. Also, clearly label any intermediate lengths as you create them. Practice on scrap paper as this page will be graded.

*This page has been left blank for your workings and solutions. Work here will not be graded. **Do not remove it!***