

THE UNIVERSITY OF BRITISH COLUMBIA

Math 335 Section 201

Calculators are allowed

No cell phones or information sheets allowed

Exam length is 2 hours and 30 minutes

FINAL EXAM

April 13, 2013

NAME

STUDENT NUMBER

[10] 1. Cantor proved that for any set S , the set $P(S)$ of subsets of S has a larger cardinality than S .

Let $S = \{A, R, I, D, G, E, H, B, T\}$ be a set of letters. Suppose you associate each element of S with a subset of S as follows:

A is associated with $\{A, D\}$

R is associated with $\{A, D, I\}$

I is associated with $\{R\}$

D is associated with $\{A, R, I, D, G\}$

G is associated with $\{\}$

E is associated with $\{D, R, E, G\}$

H is associated with $\{E, R, I\}$

B is associated with $\{R, I, B, E\}$

T is associated with $\{A, I, B, H\}$

There are many subsets of S not associated with any element of S ; however, what subset would you find using the construction that occurs in the proof of Cantor's theorem? Explain how you get your answer.

2. Three different numbers are chosen so that when each is added to the average of the remaining two the numbers 31, 33 and 34 result.

[4] (a) Find the average of the three original numbers.

[3] (b) What are the three original numbers?

[3] (c) What is the standard deviation for the three original numbers?

[Definitions. The *standard deviation* is a measure of how spread out numbers are.

The formula is easy: it is the **square root** of the **variance**. The *variance* is defined as the average of the **squared** differences from the mean and the *mean* is the simple average of the numbers.]

[10] 3. Prove that the square root of 6 is irrational.

4. [5](a) What day of the week will April 21, 2013 be? What day of the week was April 18, 2012? What day of the week will April 24, 2015 be?

[5](b) You started a long mathematics exam at noon. You were told that you could work as long as you liked. You worked 376 hours straight. At what time of day did you finish?

[10] 5. A game show requires you to randomly pick 1 of 7 envelopes. The host has hidden a \$100 bill in one envelope, and a \$1 bill in each of the other 6. Once you've picked, the host is required to remove 2 of the \$1 envelopes you didn't pick. You now have a choice of keeping your original envelope, or paying \$2 to switch your choice to one of the 4 you didn't pick.

(a) If you choose to pay \$2 and switch your choice, then what are your odds of losing money?

(b) By what percent (if any) does the odds of winning a \$100. bill change by switching your choice?

6. The frequency table of the monthly salaries of 20 people is shown below.

salary(in \$)	frequency
1200	6
2200	7
4200	4
6000	3

Note that salary \$2200 and frequency 7 means 7 people at \$2200 each.

[5](a) Calculate the mean of the salaries of the 20 people.

[5](b) Calculate the standard deviation of the salaries of the 20 people.

7. Fibonacci Nim has 2 players and a pile of sticks. Player One takes away at least 1 but not all the sticks from the pile. Player Two can take away any number from 1 to twice the number the previous player took, then player One does the same and so on. Strategy for starting player. Start with non Fibonacci number. Write as sum of non-consecutive Fibonacci numbers and discard smallest.

[5] (a) Suppose you start with 50 sticks. What is your first move?

[5] (b) Suppose you start with 33 sticks. What is the first move?

8.

[5] Solve each equation for x

(a) $x/1 = 1/(x-1)$

(b) $3x/2 = 1/(x+1)$

[5] Take rectangle and attach a square to the longer side so that you create a new larger rectangle. Suppose this new rectangle is a Golden Rectangle, then was the original rectangle a Golden one? If so, prove it. If not, show why not. [Note that a *golden rectangle* is one with ratio of longer side to shorter side of $(1+\text{square root } 5)/2$ to 1.]

[10] 9. A group of islands are connected by bridges in such a way that one can walk from any island to any other. A tourist walked around every island, crossing each bridge exactly once. He is on the island of Thrice exactly three different times. How many bridges are there to Thrice, if (a) the tourist neither started nor ended on Thrice OR (b) the tourist started on Thrice, but didn't end there OR (c) the tourist started and ended on Thrice.

10.

(5) The sailboat named Sand Bug has a tall mast. The backstay(the heavy steel cable that attaches the top of the mast to the back, or stern, of the sailboat) is made of 130 feet of cable. The base of the mast is located 50 feet from the stern of the boat. How tall is the mast?

(5) Many television screens are rectangles that are measured by the length of their diagonals. The ratio of the horizontal length to the height in a standard television screen is 4:3. The horizontal length of a “27-inch” television screen is closest, in inches, to what value?