Final Exam
Math 317
April 25th, 2017

Last Name: ____________________ First Name: ____________________

Student # : ________________ Instructor's Name : ________________

Instructions:
No memory aids allowed. No calculators allowed. No communication devices allowed. Use the space provided on the exam. If you use the back of a page, write "see back" on the front of the page. This exam is 180 minutes long.

Rules governing examinations

- Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCcard for identification.
- Candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no candidate shall be permitted to enter the examination room once the examination has begun.
- Candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
- Candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
  (a) speaking or communicating with other candidates, unless otherwise authorized;
  (b) purposely exposing written papers to the view of other candidates or imaging devices;
  (c) purposely viewing the written papers of other candidates;
  (d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
  (e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)–(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
- Candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
- Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
- Candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

Question	Points	Score
1	15	
2	11	
3	15	
4	15	
5	15	
6	16	
7	13	
Total:	100	

1. Let $\vec{\mathbf{r}}$ be the vector field $\vec{\mathbf{r}} = x\,\vec{\mathbf{i}} + y\,\vec{\mathbf{j}} + z\,\vec{\mathbf{j}}$ and let r be the function $r = |\vec{\mathbf{r}}|$. Let $\vec{\mathbf{a}}$ be the *constant* vector $\vec{\mathbf{a}} = a_1\,\vec{\mathbf{i}} + a_2\,\vec{\mathbf{j}} + a_3\,\vec{\mathbf{k}}$. Compute and simplify the following quantities. **Answers must be expressed in terms of $\vec{\mathbf{a}}$, $\vec{\mathbf{r}}$, and r. There should be no x's, y's, or z's in your answers.**

   (a) 3 points $\operatorname{div} \vec{\mathbf{r}}$

   Answer:

   (b) 4 points $\operatorname{grad}(r^2)$

   Answer:

Answer:

(c) 4 points $\operatorname{curl}(\vec{\mathbf{r}} \times \vec{\mathbf{a}})$

Answer:

(d) 4 points $\operatorname{div}\operatorname{grad}(r)$

2. (a) [3 points] Show that the planar vector field

$$\vec{\mathbf{F}}(x, y) = \langle 2xy\cos(x^2), \sin(x^2) - \sin(y)\rangle$$

is conservative.

(b) [6 points] Find a potential function for $\vec{\mathbf{F}}$.

Answer:

(c) [2 points] For the vector field $\vec{\mathbf{F}}$ from above compute $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$, where C is the part of the graph $x = \sin(y)$ from $y = \pi/2$ to $y = \pi$.

Answer:

3. A race track between two hills is described by the parametric curve

$$\vec{\mathbf{r}}(\theta) = \langle 4\cos\theta,\ 2\sin\theta,\ \frac{1}{4}\cos(2\theta)\rangle, \qquad 0 \le \theta \le 2\pi.$$

(a) 4 points Compute the curvature of the track at the point $(-4, 0, \frac{1}{4})$.

Answer:

(b) 2 points Compute the radius of the circle that best approximates the bend at the point $(-4, 0, \frac{1}{4})$ (that is, the radius of the osculating circle at that point).

Answer:

(c) 9 points A car drives down the track so that its position at time t is given by $\vec{\mathbf{r}}(t^2)$. (Note the relationship between t and θ is $\theta = t^2$). Compute the following quantities

1. The speed at the point $(-4, 0, \frac{1}{4})$.

Answer:

2. The acceleration at the point $(-4, 0, \frac{1}{4})$.

Answer:

3. The magnitude of the normal component of the acceleration at the point $(-4, 0, \frac{1}{4})$.

Answer:

4. 15 points Compute the flux integral $\iint_S \vec{\mathbf{F}} \cdot \vec{\mathbf{N}} \, dS$, where

$$\vec{\mathbf{F}} = \left\langle -\frac{1}{2}x^3 - xy^2, \, -\frac{1}{2}y^3, \, z^2 \right\rangle$$

and S is the part of the paraboloid $z = 5 - x^2 - y^2$ lying inside the cylinder $x^2 + y^2 \leq 4$, with orientation pointing downwards.

Answer:

5. Let $C = C_1 + C_2 + C_3$ be the curve given by the union of the three parameterized curves

$$\begin{aligned} \vec{\mathbf{r}}_1(t) &= \langle 2\cos t, 2\sin t, 0\rangle\,, \quad 0 \le t \le \pi/2 \\ \vec{\mathbf{r}}_2(t) &= \langle 0, 2\cos t, 2\sin t\rangle\,, \quad 0 \le t \le \pi/2 \\ \vec{\mathbf{r}}_3(t) &= \langle 2\sin t, 0, 2\cos t\rangle\,, \quad 0 \le t \le \pi/2 \end{aligned}$$

(a) [3 points] Draw a picture of C. Clearly mark each of the curves C_1, C_2, and C_3 and indicate the orientations given by the paramterizations.

(b) [5 points] Find and parameterize an oriented surface S whose boundary is C (with the given orientations).

(c) 7 points Compute the line integral $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ where

$$\vec{\mathbf{F}} = \left\langle y + \sin(x^2), z - 3x + \log(1 + y^2), y + e^{z^2} \right\rangle.$$

Answer:

6. Consider the vector field $\vec{\mathbf{F}} = P\,\vec{\mathbf{i}} + Q\,\vec{\mathbf{j}}$. Where

$$P = \frac{x+y}{x^2+y^2}, \quad Q = \frac{y-x}{x^2+y^2}.$$

(a) 3 points Compute and simplify $Q_x - P_y$

Answer:

(b) 3 points Compute the integral $\int_{C_R} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ directly using a parameterization, where C_R is the circle of radius R, centered at the origin, and oriented in the counter-clockwise direction.

Answer:

(c) 2 points Is $\vec{\mathbf{F}}$ conservative? Carefully explain how your answer fits with the results you got in the first two parts.

(d) 4 points Use Green's theorem to compute $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ where C is the triangle with vertices $(1,1),(1,0),(0,1)$ oriented in the counterclockwise direction.

Answer:

(e) 4 points Use Green's theorem to compute $\int_C \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ where C is the triangle with vertices $(-1,-1),(1,0),(0,1)$ oriented in the counterclockwise direction.

Answer:

7. Consider the vector field

$$\vec{\mathbf{F}}(x, y, z) = \langle z \arctan(y^2),\ z^3 \ln(x^2 + 1),\ 3z \rangle$$

and the surface S given by the part of the sphere $x^2 + y^2 + z^2 = 4$ that lies above the plane $z = 1$ and is oriented downwards.

(a) 3 points Find the divergence of $\vec{\mathbf{F}}$.

Answer:

(b) 10 points Compute the flux integral $\iint_S \vec{\mathbf{F}} \cdot \vec{\mathbf{N}} dS$. Hint: The surface S is not a closed surface.

Answer:

Use the space on the next page if necessary ⟶

Extra page for work.