No notes nor calculators.

Rules Governing Formal Examinations:

1. Each candidate must be prepared to produce, upon request, a UBCcard for identification;

2. Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions;

3. No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination;

4. Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action;
   (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;
   (b) Speaking or communicating with other candidates;
   (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received;

5. Candidates must not destroy or mutilate any examination material; must hand in all examination papers, and must not take any examination material from the examination room without permission of the invigilator; and

6. Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

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1. Let \( f(z) = e^{\frac{z+1}{z-1}} \). Find all \( z \) for which

(a) \(|f(z)| = 1\);  

(b) \(|f(z)| < 1\).

2. Let \( f(z) = 2x + 2y + i(x - y)^2 \) where \( z = x + iy \). Where is \( f(z) \) differentiable in the complex plane? Where is \( f(z) \) analytic? Explain your reasoning carefully.

3. Show that if \( v \) is a harmonic conjugate of \( u \) in a domain \( D \), then \( u^3 - 3uv^2 \) is harmonic in \( D \).

4. Find the radius of convergence of the Taylor series of \( f(z) = \sqrt{2} - e^z \) around \( z = 1 + 4i \). Here the square root is given by the principle branch.

5. Let \( f(z) = \frac{1}{z^2(z-3)} \). Find the maximum of \(|f(z)|\) in the annulus \( 1 \leq |z| \leq 2 \) and where it is attained.

6. All the circles below are oriented counterclockwise. Compute

(a) \( \int_{|z+1|=4} z^2 \, dz \).

(b) \( \int_{|z|=100} z^2 \sin(z^{-1}) \, dz \).

(c) \( \int_{|z|=10} \frac{\sin(3z)}{(z+2)^{10}} \, dz \).

7. Let \( f(z) = \frac{3}{(2z-1)(z-2)} \).

(a) Give the first three nonzero terms for the Laurent series of \( f(z) \) around \( \frac{1}{2} \).

(b) Give the first three nonzero terms for the Laurent series of \( f(z) \) around 0, valid for small \( |z| \). Give the region of convergence for the full expansion.

(c) Give the first three nonzero terms for the Laurent series of \( f(z) \) around 0, valid for large \( |z| \). Give the region of convergence for the full expansion.

8. Find and classify isolated singularities of \( f(z) = \frac{(\cos z - 1)e^{\frac{i}{z-1}}}{z^2(z+1)(z+2)^2} \).

9. Compute \( \int_{|z|=2} \frac{e^{2z}}{z^2(z+3)} \, dz \). Here the circle \( |z| = 2 \) is oriented counterclockwise.
Some Formulas

1. **Cauchy’s integral formula.** If \( f \) is analytic inside and on the simple closed positively oriented contour \( \Gamma \) and if \( z \) is any point inside \( \Gamma \), then

\[
f^{(n)}(z) = \frac{n!}{2\pi i} \int_{\Gamma} \frac{f(\zeta)}{(\zeta - z)^{n+1}} d\zeta \quad (n = 0, 1, 2, \ldots).
\]

2. **Laurent series.** If \( f \) is analytic in the annulus \( 0 \leq r < |z - z_0| < R \), then

\[
f(z) = \sum_{k=-\infty}^{\infty} a_k(z - z_0)^k, \quad a_k = \frac{1}{2\pi i} \int_{C} \frac{f(\zeta)}{(\zeta - z_0)^{k+1}} d\zeta, \quad (k \in \mathbb{Z}),
\]

where \( C \) is any positively oriented circle \( |z - z_0| = \rho, r < \rho < R \).

3. If \( f \) has a pole of order \( m \) at \( z_0 \), then

\[
\text{Res}(f; z_0) = \lim_{z \to z_0} \frac{1}{(m - 1)!} \left( \frac{d}{dz} \right)^{m-1} [(z - z_0)^m f(z)],
\]

that is, the \( (m - 1) \)-th Taylor coefficient of \( g(z) = (z - z_0)^m f(z) \).