The University of British Columbia
Final Examination - April 18, 2008

Mathematics 257 Section 201

Instructor: Dr. Alexei F. Cheviakov

Closed book examination | Time: 2.5 hours

Last name ______________________ **First name** ______________________

Student Number ______________________ **Signature** ______________________

Special Instructions:

- Be sure that this examination has 8 pages (5 problems). Write your last name on top of each page.

- Submit only this booklet, with solution written in space provided (you may use adjacent page(s)). Clearly outline answers. Solutions on scratch paper will not be graded.

- A 2-sided self-prepared Letter-size formula sheet is allowed. No calculators or notes are permitted.

- In case of an exam disruption such as a fire alarm, leave the exam papers in the room and exit quickly and quietly to a pre-designated location.

Rules governing examinations

- Each candidate should be prepared to produce her/his library/AMS card upon request.
- No candidate shall be permitted to enter the examination room after the expiration of one half hour, or to leave during the first half hour of examination.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- CAUTION - Candidates guilty of any of the following or similar practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  (a) Making use of any books, papers, or memoranda, other than those authorized by the examiners.
  (b) Speaking or communicating with other candidates.
  (c) Purposely exposing written papers to the view of other candidates.
- Food and smoking are not permitted during examinations.

1		20
2		20
3		20
4		20
5		20
Total		100

[20] **Problem 1.**

a) Find the coefficients of Fourier cosine series of the function $f(x) = \begin{cases} 1, & 0 \le x \le 2, \\ 4 - x, & 2 < x \le 4. \end{cases}$

b) Write the series in the form where coefficients do not contain trigonometric functions.

c) What is the period of the series you found? Sketch the graph of the function to which the series converges over at least two periods.

d) Does the series converge uniformly or pointwise? At which points x does Gibbs phenomenon occur?

[20] **Problem 2.** Solve the following problem for the Laplace equation in the rectangle:

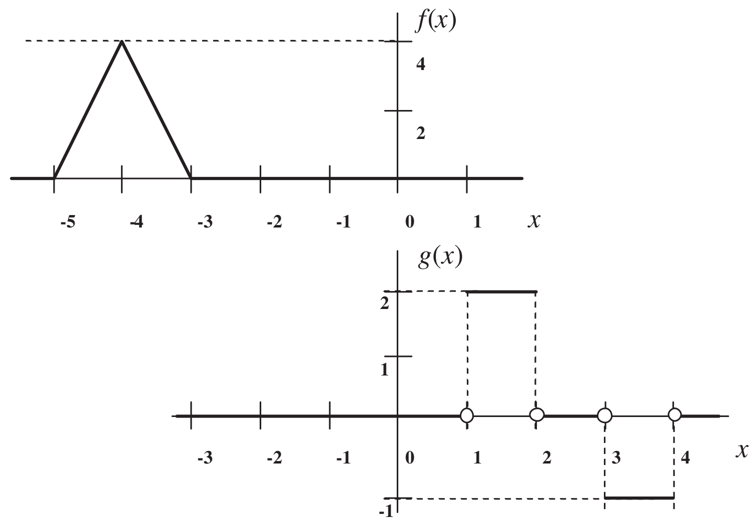
$$\begin{cases} \Delta u(x,y) = 0, \quad 0 < x < 4, \quad 0 < y < 3, \\ u_x(0,y) = u_x(4,y) = u_y(x,0) = 0, \\ u(x,3) = f(x), \end{cases}$$

where $f(x)$ is given in Problem 1. [**Remark:** you do not have to do all steps of separation of variables explicitly. You may state results you know, e.g., eigenbasis, without derivation.]

[20] **Problem 3.** Consider the wave equation $u_{tt} = u_{xx}$ for an infinite string $x \in (-\infty, \infty)$ with initial conditions $u(x,0) = f(x), \quad u_t(x,0) = g(x)$.

a) Give the formula for $u(x,t)$ for all $t > 0$, $-\infty < x < +\infty$.

b) Draw the solution $u(x,t)$ for $t = 2$, if $f(x)$ and $g(x)$ are functions given below:



[Hint: consider separately a problem with $f = 0$ and a problem with $g = 0$.]

c) Now suppose the string is finite: $-10 \le x \le 10$. Specify the maximum time T until which your solution found in a) for the infinite string, is correct for this finite string.

[20] **Problem 4.** Consider a Sturm-Liouville problem

$$\begin{cases} X'' + \lambda X = 0, \quad 0 < x < 1, \\ X(0) = 0, \\ X(1) - X'(1) = 0. \end{cases}$$

a) Is the problem regular or singular? Why? (Use the definition.)

b) Solve the problem to find all its eigenfunctions and eigenvalues.

c) Write down the orthogonality condition satisfied by the eigenfunctions. Find norms of all eigenfunctions.

[20] **Problem 5.** Consider the following dimensionless problem, describing lengthwise oscillations of an elastic bar with forcing and friction:

$$\begin{cases} u_{tt} + 2u_t = u_{xx} + x, \quad 0 < x < 1,\ 0 < t, \\ u(0,t) = 0, \\ u(1,t) - u_x(1,t) = 0, \\ u(x,0) = 1, \\ u_t(x,0) = 0. \end{cases}$$

Solve this problem using separation of variables ideas. [**Hint:** Start from a homogeneous version of the equation. Use the results of Problem 4.]

[extra paper]

[extra paper]