1) Show that \( x = 0 \) is a regular singular point of the differential equation \( x^2y'' + xy' - x^2y = 0 \).

2) Find two linearly independent solutions of the differential equation near \( x = 0 \). What is the radius of convergence of the two power series solutions?

II-[15] Let

\[
f(x) = \begin{cases} 
\pi & \text{if } -\pi \leq x < 0 \\
\pi - x & \text{if } 0 \leq x < \pi 
\end{cases}
\]

with \( f(x + 2\pi) = f(x) \).

1) Find the Fourier series corresponding to the given function \( f(x) \).

2) Sketch the graph of the function to which the series converges over two periods.

III-[25] Solve the damped wave equation

\[
\begin{align*}
\frac{\partial^2 u}{\partial t^2} + 2\frac{\partial u}{\partial t} &= \frac{\partial^2 u}{\partial x^2} - \sin x, & 0 < x < \pi, \ 0 < t \\
u(0, t) &= 0, & u_x(\pi, t) = 2 \\
u(x, 0) &= f(x), & u_t(x, 0) = 0, \ 0 < x < \pi.
\end{align*}
\]

Hint Write \( u(x, t) = v(x) + w(x, t) \).
IV-[25] Solve the boundary value problem
\[ u_{xx} + u_{yy} + 2u_y + u = 0, \quad 0 < x < 1, \quad 0 < y < 1 \]
\[ u(0, y) = 0 = u(1, y), \quad 0 < y < 1 \]
\[ u(x, 0) = 3 \sin(5\pi x), \quad u_y(x, 1) = 2 \sin(3\pi x) \]

V-[20] Solve the initial boundary value problem
\[ u_t - u_{xx} = t \sin(5\pi x), \quad 0 < x < 1, \quad 0 < t \]
\[ u(0, t) = 0 = u(1, t) \]
\[ u(x, 0) = x(1 - x), \quad 0 < x < 1. \]