The University of British Columbia

Final Examination - December 16, 2008

Mathematics 230

Section 101

Closed book examination Time: 2.5 hours

Last Name _____ **First** _____ **Signature** _____

Student Number _____

Special Instructions:

No notes or calculators are allowed. Answer all 6 questions on the sheets provided - use the backs of the sheets if necessary.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
  (b) Speaking or communicating with other candidates.
  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		31
2		10
3		12
4		12
5		10
6		15
Total		90

[31] **1**. Arithmetic

1. (4pts) Express as a product of primes:

   (a) $(27 * 17)^2$

   (b) $(121)^3$

2. (4pts) Compute the greatest common divisor and the least common multiple for the pair of integers $(108, 180)$. You may leave your answer as a product of primes.

3. (2pts)Determine the integer nearest to the given fraction.

$$\frac{201}{23}$$

4. (3pts) Evaluate, eliminating parentheses first.

$$-\frac{3}{8}+\left(\frac{1}{2}-\frac{5}{6}\right)-\left(\frac{7}{4}-\frac{1}{3}\right)-\frac{7}{6}+\frac{1}{8}$$

5. (3pts) Evaluate

$$\left[\left(\frac{2}{5}\right)^4\times\left(\frac{2}{5}\right)^3\right]^2\div\left(\frac{2}{5}\right)^{12}$$

6. (10pts) Simplify

(a) $\left(\sqrt{2}\right)^5$

(b) $\sqrt{15}\sqrt{65}$

(c) $6\sqrt{48} - 7\sqrt{27}$

(d) $\sqrt{3\sqrt{5}}\sqrt{5\sqrt{3}}\sqrt{\sqrt{6}}$

7. (5pts) Evaluate

$$\frac{5}{30} - \frac{1}{7} \times \left[\left(4 - \frac{18}{27} \right) \div \frac{8}{3} + \left(\frac{3}{8} \right)^2 \times \left(\frac{7}{4} \times \frac{7}{9} + \frac{5}{12} \right) \right]$$

[10] **2**.

(a) If these letters, as printed here, were made from perfectly thin lines, then for which pair or pairs, is the uppercase letter equivalent by distortion to its lower case partner? A brief response without explanation is sufficient here.

A,a B,b C,c R,r

(b) Are the following letters equivalent to each other by distortion? Explain.

(i) X A

(i) Θ T

[12] **3**.

(a) Is it possible to draw a connected graph with 14 vertices and 12 edges? Explain

(b) Is it possible to draw a connected graph in the plane with 8 vertices, 10 edges and 5 faces. Explain

(c) In considering the Euler Characteristic in class, we looked at connected graphs drawn in the plane. Suppose we now consider a graph that has exactly two pieces; that is, this graph is not connected but instead consists of two connected parts that do not touch each other. For any such graph, what would $V - E + F$ equal? Explain.

Illustrate your answer with an example.

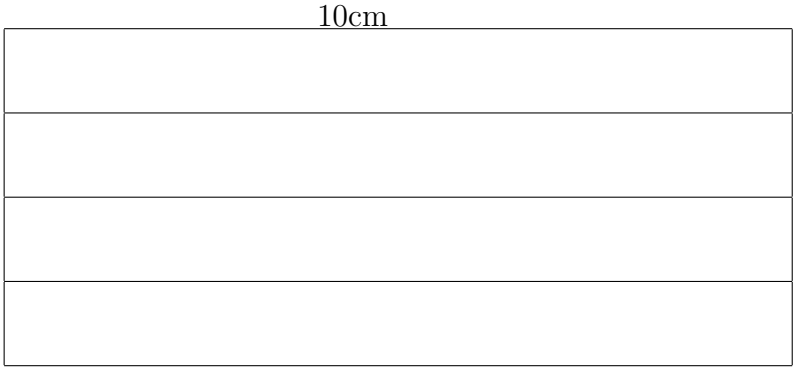
[12]**4**. If you took the rectangle below and joined the right and left edges with a twist, you would obtain a Möobius band. Suppose you cut the Möbius band on the lines drawn.

Please, answer the following questions and explain your reasoning.

(a) How many pieces shall you get?

(b) How many edges does each piece have? How many edges did the original Möbius band have?

(c) Suppose the long side of the original rectangle is 10cm. Find lengths of edges of the original Möbius band and the pieces you obtained after cutting along the drawn lines.



[10] **5**. Suppose voters rank possible candidates as follows.

	Candidate A	Candidate B	Candidate C
First choice votes	5	6	4
Second choice votes	4	2	9
Third choice votes	6	7	2

Determine the winner using plurality, vote-for-two, and Borda count voting methods. Explain your reasoning in each case.

[15] **6**. Write on one of the following topics.

(a) *"Mathematics consists in proving the most obvious thing in the least obvious way."* George Polya

Most people have an idea of what mathematics is, even though a definition is hard to come by. What do you think mathematics is? How have your ideas changed over the past semester? Comment on this with references to ideas from the class to support your statements.

(b) *"By and large it is uniformly true that in mathematics there is a time lapse between a mathematical discovery and the moment it becomes useful; and that this lapse can be anything from 30 to 100 years, in some cases even more; and that the whole system seems to function without any direction, without any reference to usefulness, and without any desire to do things which are useful."* John von Neumann

What do you think motivates mathematical explorations? Why do you think mathematics has turned out to be so useful in so many areas? Comment on this, with references to ideas from the class and the readings to support your statements.