

Marks

[10] 1. A curve in  $\mathbf{R}^3$  is given by the parametric equation  $\mathbf{x}(t) = (e^t, e^{-t}, \sqrt{2}t)$ .

(a) (4 marks) Find the length of the curve between  $t = 0$  and  $t = 1$ .

(b) (6 marks) Find the curvature at a general point  $\mathbf{x}(t)$ .

[12] **2.** Let  $f(x, y) = xe^y - y^2e^x$ .

(a) (6 marks) Find the first- and second-order Taylor polynomials  $P_1(x, y)$  and  $P_2(x, y)$  at  $(0, 1)$ . (It is not necessary to simplify your answers.)

(b) (6 marks) Prove that  $f(x, y) - P_1(x, y)$ , where  $P_1$  is the first-order Taylor polynomial from (a), has a local maximum at  $(0, 1)$ . (Hints: (1)  $e \approx 2.72$ , (2) you have done most of the necessary calculations in (a).)

- [8] **3.** Find the minimum distance from the origin to the surface  $z(3x + 4y) = 20$ .

[10] 4. Evaluate the following integrals:

(a) (5 marks)  $\iint_D \cos(x^2) dA$ , where  $D$  is the triangle in the  $xy$ -plane with vertices  $(0, 0)$ ,  $(2, 0)$ ,  $(2, 2)$ ;

(b) (5 marks)  $\iiint_W xz dV$ , where  $W$  is the bounded solid enclosed by the planes  $z = 0$ ,  $z = 2$ ,  $y = 0$ ,  $y = x$ , and the cylinder  $x^2 + y^2 = 1$ .

[12] 5. Evaluate the line integrals below. (Use any methods you like.)

(a) (6 marks)  $\int_{\mathbf{x}} \mathbf{F} \cdot d\mathbf{s}$ , where  $\mathbf{F} = x\mathbf{i} + 2y\mathbf{j} + 4z\mathbf{k}$  and  $\mathbf{x}$  is the parametrized curve  $(\cos t + \sin t, \cos t - \sin t, t)$ ,  $0 \leq t \leq 1$ .

(b) (6 marks) The (outward) flux of  $\mathbf{F}(x, y) = (x^3 + \sin y)\mathbf{i} + e^{x+y}\mathbf{j}$  across the boundary of the rectangle  $0 \leq x \leq 1$ ,  $0 \leq y \leq 2$  in the  $xy$ -plane.

[12] **6.**

(a) (4 marks) Find a function  $f(x, y)$  such that  $\mathbf{F} = \nabla f$ , where  $\mathbf{F}(x, y) = (x^2 + y^2)\mathbf{i} + 2xy\mathbf{j}$ .

(b) (4 marks) Evaluate  $\int_C \mathbf{F} \cdot d\mathbf{s}$ , where  $C$  is any oriented piecewise  $C^1$  curve from  $(1, 2)$  to  $(3, 4)$  and  $\mathbf{F}$  is the vector field in (a).

(c) (4 marks) Let  $\mathbf{F} = \nabla f$  be a conservative vector field (not necessarily the same as in (a)-(b)), and let  $\mathbf{x}(t)$  be a flow line of  $\mathbf{F}$ . Prove that  $\frac{d}{dt}f(\mathbf{x}(t)) \geq 0$ .

- [8] 7. Let  $\mathbf{F} = (x + z)\mathbf{i} + (y + 2z)\mathbf{j} + (2x + 3y)\mathbf{k}$ . What are the possible values of  $\int_C \mathbf{F} \cdot d\mathbf{s}$ , if  $C$  is a circle of radius  $r$  contained in a plane  $x + 3y - z = a$ ?

- [16] 8. Let  $\mathbf{X}$  be the parametrized surface  $\mathbf{X}(s, t) = (t \cos s, t \sin s, 2t)$ ,  $0 \leq s \leq \pi/2$ ,  $1 \leq t \leq 2$ . Evaluate the following integrals:

(a) (8 marks)  $\int \int_{\mathbf{X}} z^2 dS$ ,

(b) (8 marks)  $\int \int_{\mathbf{X}} \mathbf{F} \cdot d\mathbf{S}$ , if  $\mathbf{F} = y^2 \mathbf{i}$ .

[12] 9. Let  $\omega = (x + z)dx \wedge dy + (y - x)dy \wedge dz$ .

(a) (4 marks) Compute  $d\omega$ . Simplify your answer.

(b) (8 marks) Find  $\int_{\mathbf{X}} \omega$ , if  $\mathbf{X}(s, t) = (t + s, t, s^2)$ ,  $-1 \leq s \leq 1$ ,  $0 \leq t \leq 1$ .

Be sure that this examination has 10 pages including this cover

The University of British Columbia

Sessional Examinations - April 2009

Mathematics 227

Advanced Calculus II

Closed book examination

Time: 2.5 hours

Print Name \_\_\_\_\_ Signature \_\_\_\_\_

Student Number \_\_\_\_\_ Instructor's Name \_\_\_\_\_

Section Number \_\_\_\_\_

**Special Instructions:**

No calculators, notes, or books of any kind are allowed.  
Show all calculations for your solutions. If you need more space than is provided, use the back of the previous page.

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Total		100