Math 221: Final Exam

Date: December 6, 2008

Name (please print):
Student Number:
Section number:
Instructions : No textbook, notes, or other aids allowed. Show all your work. If you need more space, use the back of the page. Each problem is worth 10

There are ${\bf 14}$ pages in this exam. The last 3 pages are blank for scratch work. Please return all 14 pages.

marks (5+5).

Problem 1a: [5] Let A denote the matrix $A = \begin{pmatrix} 3 & 0 & -2 \\ 1 & -2 & 3 \\ 4 & -2 & 1 \end{pmatrix}$. Find a basis for the column space of A.

Problem 1b: [5] Determine whether the vector $b = \begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$ is in the column space of the matrix A. above.

Problem 2a: [5] Let $A = \begin{pmatrix} 1 & 0 & -2 \\ 1 & 1 & 0 \\ 0 & -2 & h \end{pmatrix}$. Find all values of h such that A is NOT invertible.

Problem 2b: [5] True or false (explain your answer): if A is an $n \times n$ matrix such that $\lambda = 0$ is an eigenvalue of A, then A is not invertible.

Problem 3: [5] Show that the determinant of the matrix $A = \begin{pmatrix} 1 & x & x^2 \\ 1 & y & y^2 \\ 1 & z & z^2 \end{pmatrix}$ is given by (z-x)(y-x)(z-y).

Problem 3b: [5] Find the determinant of the matrix $A = \begin{pmatrix} 3 & 3 & 3 \\ 1 & 5 & 25 \\ 10 & 70 & 490 \end{pmatrix}$. (Hint: you can use part (a) above to save some calculation.)

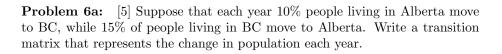
Problem 4a: [5] Let
$$W$$
 be the plane in \mathbb{R}^4 spanned by the vectors $v = \begin{pmatrix} 1 \\ 1 \\ 1 \\ 1 \end{pmatrix}$

and
$$w = \begin{pmatrix} 2 \\ 0 \\ 2 \\ 0 \end{pmatrix}$$
. Verify that the vector $u = \begin{pmatrix} 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$ is in the subspace W and show that the vectors u and w form an orthogonal basis for W .

Problem 4b: [5] Find the vector in
$$W$$
 which is closest to $b = \begin{pmatrix} 1 \\ 2 \\ 3 \\ 4 \end{pmatrix}$

Problem 5a: [5] Find the eigenvalues and eigenvectors for $A = \begin{pmatrix} 5 & 0 \\ 2 & 1 \end{pmatrix}$.

Problem 5b: [5] Find the matrix A^{1000} . (Simplify your answer as much as possible.)



Problem 6b: [5] Find limiting population distribution if in the initial year, there at 100,000 people living in each province BC and Alberta.

Problem 7a: [5] Find the characteristic polynomial of the matrix $\begin{pmatrix} 1 & 0 & -1 \\ 2 & 3 & -1 \\ 0 & 6 & 0 \end{pmatrix}$. Give your answer in the form $-\lambda^3 + a\lambda^2 + b\lambda + c$ where a,b,c are real numbers.

Problem 7b: [5] True or false (explain your answer): If A and B are similar matrices, then A and B have the same eigenvalues.

Problem 8a: [5] Suppose A and B are square matrices with AB = BA, and B invertible. Then if v is an eigenvector of A with eigenvalue λ , show that Bv is an eigenvector of A with the same eigenvalue.

Problem 8b: [5] True or false (explain your answer): let T be the transformation of $\mathbf{R}^2 \to \mathbf{R}^2$ given by $T(\begin{pmatrix} x \\ y \end{pmatrix}) = \begin{pmatrix} x+y \\ x-y \end{pmatrix}$. Then T is a linear transformation.

Problem 9a: [5] Let $B = \{b_1, b_2\}$ be a basis for \mathbf{R}^2 and let T be the linear transformation $\mathbf{R}^2 \to \mathbf{R}^2$ such that $T(b_1) = 2b_1 + b_2$ and $T(b_2) = b_2$. Find the matrix of T relative to the basis B.

Problem 9b: [5] Suppose now that $b_1 = \begin{pmatrix} 1 \\ 1 \end{pmatrix}$ and $b_2 = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$. Find the matrix of T relative to the standard basis of \mathbf{R}^2 .

Problem 10a: [5] True or false (explain your answer): Suppose that v_1, \ldots, v_n are a basis for \mathbf{R}^n , and A is an invertible $n \times n$ matrix. Then the vectors Av_1, \ldots, Av_n are also a basis for \mathbf{R}^n .

Problem 10b: [5] True or false (explain your answer): if A is a 2×2 matrix with characteristic polynomial $(\lambda - 2)^2$ then it is diagonalizable.