

Math 217 Final Exam, December 2013

Last Name: _____ First Name: _____

Student Number: _____ Signature: _____

Instructions. The exam lasts 2.5 hours. **No calculators or electronic devices of any kind are permitted.** There are **10 pages** in this test including this cover page and blank pages. Unless otherwise indicated, show all your work.

Rules governing examinations

- Each examination candidate must be prepared to produce, upon the request of the invigilator or examiner, his or her UBCCard for identification.
- Candidates are not permitted to ask questions of the examiners or invigilators, except in cases of supposed errors or ambiguities in examination questions, illegible or missing material, or the like.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination. Should the examination run forty-five (45) minutes or less, no candidate shall be permitted to enter the examination room once the examination has begun.
- Candidates must conduct themselves honestly and in accordance with established rules for a given examination, which will be articulated by the examiner or invigilator prior to the examination commencing. Should dishonest behaviour be observed by the examiner(s) or invigilator(s), pleas of accident or forgetfulness shall not be received.
- Candidates suspected of any of the following, or any other similar practices, may be immediately dismissed from the examination by the examiner/invigilator, and may be subject to disciplinary action:
 - (a) speaking or communicating with other candidates, unless otherwise authorized;
 - (b) purposely exposing written papers to the view of other candidates or imaging devices;
 - (c) purposely viewing the written papers of other candidates;
 - (d) using or having visible at the place of writing any books, papers or other memory aid devices other than those authorized by the examiner(s); and,
 - (e) using or operating electronic devices including but not limited to telephones, calculators, computers, or similar devices other than those authorized by the examiner(s)-(electronic devices other than those authorized by the examiner(s) must be completely powered down if present at the place of writing).
- Candidates must not destroy or damage any examination material, must hand in all examination papers, and must not take any examination material from the examination room without permission of the examiner or invigilator.
- Notwithstanding the above, for any mode of examination that does not fall into the traditional, paper-based method, examination candidates shall adhere to any special rules for conduct as established and articulated by the examiner.
- Candidates must follow any additional examination rules or directions communicated by the examiner(s) or invigilator(s).

Problem #	Value	Grade
1	7	
2	8	
3	8	
4	10	
5	9	
6	8	
Total	50	

1. Consider the function $f(x, y) = e^{xy} + 2x \cos(y)$.
 - (a) Compute its first and second partials: f_x , f_y , f_{xx} , f_{xy} , f_{yy} and f_{yx} .
 - (b) Find an equation for the tangent plane to its graph at $(x, y) = (1, 0)$.
 - (c) Use the linear approximation at $(x, y) = (1, 0)$ to estimate $f(1.02, 0.02)$.

2. Find the centroid (the centre of mass when the density is constant) of the solid region bounded by the surfaces $z = (x^2 + y^2)^2$ and $z = 2 - x^2 - y^2$.

3. Let E be the solid region bounded by the planes $x = 0$, $y = 0$, $x = 1$, and $y = 1$. Let C be the curve of intersection of the surface $z = y^2$ with the boundary of E , oriented counterclockwise when viewed from above. Let

$$\mathbf{F}(x, y, z) = (x - z^3)\hat{\mathbf{i}} + (3xz^2 + y)\hat{\mathbf{j}} + (y + z)\hat{\mathbf{k}}.$$

Use Stokes' theorem to calculate $\int_C \mathbf{F} \cdot d\mathbf{r}$.

4. A rectangular box has two opposing sides (left and right) made of gold, two (front and back) of silver, and two (top and bottom) of bronze. The cost (per unit area) of the gold is $\$4/m^2$, the silver is $\$2/m^2$, and the bronze is $\$1/m^2$. The box has volume $8 m^3$, and each of its edges is no more than $8 m$ long. What are the minimum and maximum possible costs of such a box?

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5. Consider the 2D vector field

$$\mathbf{F}(x, y) = \frac{-y}{x^2 + y^2} \hat{\mathbf{i}} + \frac{x}{x^2 + y^2} \hat{\mathbf{j}} = P(x, y) \hat{\mathbf{i}} + Q(x, y) \hat{\mathbf{j}}$$

defined on $D = \{(x, y) \in \mathbb{R}^2 \mid (x, y) \neq (0, 0)\}$.

- (a) Let C_1 be the square of side length 2 centred around the origin (i.e. with corners $(\pm 1, \pm 1)$), with counter-clockwise orientation. Compute the line integral $\int_{C_1} \mathbf{F} \cdot d\mathbf{r}$.
Hint: it may help to recall $\int \frac{du}{1+u^2} = \tan^{-1}(u) + C$.
- (b) Let C_2 be the circle of radius 2, $x^2 + y^2 = 4$, with counter-clockwise orientation. Compute the line integral $\int_{C_2} \mathbf{F} \cdot d\mathbf{r}$.
- (c) Now you are given that $Q_x(x, y) = P_y(x, y)$ on D (it is true – you do not have to check it!). Your answers to (a) and (b) should be the same; explain why, using Green's theorem.
- (d) Is \mathbf{F} conservative on D ? If not, explain why not, given that $Q_x = P_y$. If so, find a potential function.
- (e) Is \mathbf{F} conservative on the disk of radius 1 centred at $(2, 2)$? If not, explain why not, given that $Q_x = P_y$. If so, find a potential function.

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6. Let S be the ellipsoid $x^2/a^2 + y^2/b^2 + z^2/c^2 = 1$ (a, b, c are positive constants).

(a) Find a parameterization of S , and use it to express the surface integral

$$\iint_S \left(\frac{x^2}{a^4} + \frac{y^2}{b^4} + \frac{z^2}{c^4} \right)^{1/2} dS$$

as a double integral (but do not try to evaluate the double integral).

(b) Evaluate the surface integral above by using the Divergence Theorem.

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