The University of British Columbia

Final Examination - December 16, 2011

Mathematics 217

Time: 2.5 hours

LAST Name ______________________________

First Name ____________________ **Signature** ____________________

Student Number ________________

Special Instructions:

One formula sheet allowed. No communication devices allowed. One calculator allowed. Show all your work; little or no credit will be given for a numerical answer without the correct accompanying work. If you need more space than the space provided, use the back of the previous page.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.
  (b) Speaking or communicating with other candidates.
  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		10
2		10
3		10
4		10
5		10
6		10
Total		60

[10] **1**. Recall that a direction can be given by a unit vector of the form $\vec{\mathbf{u}}_\theta = \langle \cos\theta, \sin\theta \rangle$ for some angle θ measured counterclockwise from the positive x-axis. Let

$$f(x, y) = e^{-x^2-2y^2},$$

and consider the point $P(-1, 0)$.

(a) Find ∇f at P.

(b) Find the angles θ that determine the directions of maximum increase, maximum decrease, and zero change of the function f.

(c) Write the directional derivative of f at P as a function of θ; call this function $g(\theta)$.

(d) Find the value of θ that maximizes $g(\theta)$ and find the maximum value.

(e) Verify that the value of θ that maximizes g corresponds to the direction of the gradient. Verify that the maximum value of g equals the magnitude of the gradient.

[10] **2**. Consider the parabola $y = x^2$, which we parametrize as $\vec{\mathbf{r}}(t) = \langle t, t^2 \rangle$ for $-\infty < t < \infty$.

(a) Find the unit tangent and unit normal vectors $\vec{\mathbf{T}}(t)$ and $\vec{\mathbf{N}}(t)$, respectively, for this parabola.

(b) Show that the curvature of the parabola $y = x^2$ at its vertex is $\kappa = 2$.

(c) Find the equation of the osculating circle at the vertex of the parabola. Sketch both the parabola and this osculating circle on the same set of axes.

[10] **3**. Evaluate the triple iterated integral

$$\int_{-3}^{3}\int_{0}^{\sqrt{9-x^2}}\int_{0}^{2}\frac{1}{1+x^2+y^2}\,dz\,dy\,dx.$$

It will be useful to sketch the region in $\mathbb{R}^3$ over which you are integrating.

[10] **4**. Let C_1 be the circle $(x-2)^2 + y^2 = 1$ and let C_2 be the circle $(x-2)^2 + y^2 = 9$. Suppose

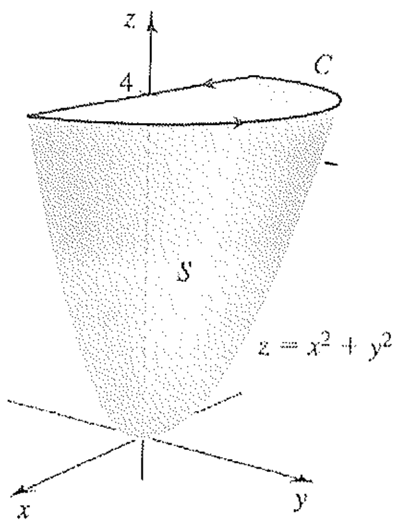
$$\vec{\mathbf{F}}(x, y) = \left\langle \frac{-y}{x^2+y^2}, \frac{x}{x^2+y^2} \right\rangle.$$

Find the integrals $\int_{C_1} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$ and $\int_{C_2} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$, where both circles are oriented counterclockwise in computing the line integrals.

[10] 5. Consider the surface S that is the portion of the paraboloid $z = x^2 + y^2$ with $0 \leq z \leq 4$ and $y \geq 0$ together with the planar surface bounded by the parabola $z = x^2$ in the xz-plane. Let C be the semicircle and line segment that bound the cap of S in the plane $z = 4$ with counterclockwise orientation. Let $\vec{\mathbf{F}}(x, y, z) = \langle 2z + y, 2x + z, 2y + x \rangle$.

Find $\iint_{S_1} \nabla \times \vec{\mathbf{F}} \cdot \vec{\mathbf{n}}\, dS$ where S_1 is the portion of the surface S formed by the half of the paraboloid with $y \geq 0$ and $0 \leq z \leq 4$.

Hint: Use Stokes' theorem, but apply it to the original surface S.



[Q5 continued]

[10] **6**. Find $\iint_S \vec{\mathbf{F}} \cdot \vec{\mathbf{n}}\; dS$, the flux of the vector field $\vec{\mathbf{F}}(x, y, z) = \langle x \sin y, -\cos y, z \sin y \rangle$ across the surface S, where S is the boundary of the region in $\mathbb{R}^3$ bounded by the planes $x = 1,\ y = 0,\ y = \pi/2,\ z = 0$, and $z = x$.

The End