[12] **1.** Consider the surface $S: \quad \cos(\pi x) - x^2 y + e^{xz} + yz = 4.$

(a) Find the plane tangent to S at $(0, 1, 2)$.

(b) Suppose $(0.03, 0.96, z)$ lies on S. Give an approximate value for z.

(c) Suppose $a > 0$ is very small. Then the circular cylinder $x^2 + (y-1)^2 = a^2$ cuts a tiny disk from the surface S. Approximately what is the area of this disk?

[12] **2.** Show that each critical point of this function gives a local minimum:

$$f(x, y) = \tfrac{1}{2}(x^2 y - x - 1)^2 + \tfrac{1}{2}(x^2 - 1)^2.$$

[12] **3.** Find the centroid $(\overline{x}, \overline{y}, \overline{z})$ of the solid inside the cylinder $x^2 + y^2 = 4$, above the plane $z = 0$, and below the paraboloid $z = 1 + x^2 + y^2$.

[12] **4.** Let $\quad I = \displaystyle\int_1^2 \int_{-y}^{y/\sqrt{3}} \frac{1}{\sqrt{x^2 + y^2}} \, dx \, dy.$

(a) Rewrite I as an iterated integral in polar coordinates.

(b) Evaluate I.

Hints: $\quad \displaystyle\int \sec(at)\, dt = a^{-1} \ln|\sec(at) + \tan(at)|, \qquad \int \csc(at)\, dt = a^{-1} \ln|\csc(t) - \cot(t)|.$

[12] **5.** Let C be a simple closed curve in the plane $2x + 2y + z = 2$, oriented counterclockwise when viewed from high on the z-axis.

(a) Show that

$$I(C) \stackrel{\text{def}}{=} \oint_C 2y\, dx + 3z\, dy - x\, dz$$

depends only on the area of the region enclosed by C and not on the position or shape of C.

(b) Let C be the triangular path from $(1, 0, 0)$ to $(0, 1, 0)$ to $(0, 0, 2)$ to $(1, 0, 0)$. Find $I(C)$ by calculating a cross product and using part (a).

[12] **6.** Let S be the surface cut from the parabolic cylinder $z = 1 - y^2$ by the planes $x = 0$, $x = 3$, and $z = 0$. Evaluate

$$I_2 = \iint_S \frac{y^2 z}{\sqrt{4y^2 + 1}}\, dS \qquad \text{and} \qquad I_3 = \iint_S \frac{y^3 z}{\sqrt{4y^2 + 1}}\, dS.$$

[12] **7.** For each $a > 0$, evaluate $\quad I_a \stackrel{\text{def}}{=} \displaystyle\int_{C_a} \Big(e^x \ln(y)\Big)\, dx + \left(\frac{e^x}{y} + \sin(z)\right) dy + \Big(y\cos(z)\Big)\, dz, \quad$ given

$$C_a: \qquad x = a\cos(t), \quad y = a, \quad z = a\sin(t), \qquad 0 \le t \le \pi.$$

[12] **8.** A particle travels from $(1, 2)$ to $(-1, 2)$ along the curve $y = 3 - x^2$, then back to $(1, 2)$ along the curve $y = x^4 + 1$, under the influence of the force

$$\mathbf{F} = (y + e^x \ln(y))\, \mathbf{i} + (e^x / y)\, \mathbf{j}.$$

Find the work done, i.e., $\quad W = \displaystyle\oint_C \mathbf{F} \bullet d\mathbf{r}, \quad$ for the curve C described above.

Continued on page 3

[12] **9.** Let S denote the part of the surface $z = e^{-x^2}$ selected by the simultaneous inequalities $y \geq 0$, $x \leq 1$, $y \leq x$, and let

$$\mathbf{F} = \left\langle x^2y - xy,\ xy^2 - xy,\ z(1 + x + y - 4xy) \right\rangle .$$

Let Φ be the upward flux of $\mathbf{F}$ through S.

(a) Express Φ as a double integral over a suitable region D in xy-space.

(b) Use the Divergence Theorem to express Φ as a different double integral over D. Suggestion: Imagine S as the top surface of a solid E, whose bottom is $z = 0$ and whose sides are vertical planes.

(c) Evaluate Φ.

The End

This examination has 11 pages including this cover

The University of British Columbia
Sessional Examination – December 2005

Mathematics 217
Multivariable and Vector Calculus

Closed book examination | Time: $2\frac{1}{2}$ hours

Name____________________ **Signature** ____________________

Student Number____________________

Special Instructions:

Calculators may NOT be used.
A formula sheet has been provided.
If you need more space than is provided for a question, use the back of the previous page.

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1		12
2		12
3		12
4		12
5		12
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7		12
8		12
9		12
Total		108