The University of British Columbia

Final Examination - April 17, 2009

Mathematics 200

Instructors: Dr. Keqin Liu (Sec. 201) and Dr. Dale Peterson (Sec. 202)

Closed book examination | Time: 2.5 hours

Last Name: ____________________, **First:** ______________ **Signature** ____________________

Student Number ____________________

Special Instructions:

No books, notes or calculators are allowed.

Rules governing examinations

- Each candidate must be prepared to produce, upon request, a UBCcard for identification.
- Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
- No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
- Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.

  (a) Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners.

  (b) Speaking or communicating with other candidates.

  (c) Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.
- Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.
- Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

1		14
2		10
3		10
4		16
5		10
6		10
7		20
8		10
Total		100

[14] **1**. Let $f(x, y) = \dfrac{x^2 y}{x^4 + 2y^2}$.

(i) Find the tangent plane to the surface $z = f(x, y)$ at the point $\left(-1, 1, \dfrac{1}{3}\right)$.

(ii) Find an approximate value for $f(-0.9, 1.1)$.

[10] **2**. Let $f(x)$ and $g(x)$ be two functions of x satisfying $f''(7) = -2$ and $g''(-4) = -1$. If $z = h(s,t) = f(2s+3t) + g(s-6t)$ is a function of s and t, find the value of $\frac{\partial^2 z}{\partial t^2}$ when $s = 2$ and $t = 1$.

[10] **3**. Let $f(x, y) = 2x^2 + 3xy + y^2$ be a function of x and y.

(i) Find the maximum rate of change of $f(x, y)$ at the point $P\left(1, -\frac{4}{3}\right)$.

(ii) Find the directions in which the directional derivative of $f(x, y)$ at the point $P\left(1, -\frac{4}{3}\right)$ has the value $\frac{1}{5}$.

[16] **4**. Use Lagrange multipliers to find the maximum and minimum values of the function $f(x, y, z) = x^2 + y^2 - \frac{1}{20}z^2$ on the curve of intersection of the plane $x + 2y + z = 10$ and the paraboloid $x^2 + y^2 - z = 0$.

[10] **5**. Let I be the double integral of the function $f(x, y) = y^2 \sin xy$ over the triangle with vertices $(0, 0)$, $(0, 1)$ and $(1, 1)$ in the xy-plane.

(i) Write I as an iterated integral in two different ways. [5 pts]

(ii) Evaluate I. [5 pts]

[10] **6**. Evaluate $\displaystyle\iint\limits_{\mathbb{R}^2} \frac{1}{(1+x^2+y^2)^2}\, dA.$

[20] **7**. A solid is bounded below by the cone $z = \sqrt{x^2 + y^2}$ and above by the sphere $x^2 + y^2 + z^2 = 2$. It has density $\rho(x, y, z) = x^2 + y^2$.

(i) Express the mass M of the solid as a triple integral, with limits, in cylindrical coordinates. [5 pts]

(ii) Same as (i) but in spherical coordinates. [5 pts]

(iii) Evaluate M. [10 pts]

[10] **8**. Let $I = \iiint\limits_E f(x, y, z)\, dV$, where E is the tetrahedron with vertices $(-1, 0, 0)$, $(0, 0, 0)$, $(0, 0, 3)$ and $(0, -2, 0)$.

(i) Rewrite the integral I in the form

$$I = \int_{x=}^{x=} \int_{y=}^{y=} \int_{z=}^{z=} f(x, y, z)\, dz\, dy\, dx.$$

(ii) Rewrite the integral I in the form

$$I = \int_{z=}^{z=} \int_{x=}^{x=} \int_{y=}^{y=} f(x, y, z)\, dy\, dx\, dz.$$