Math 200 - Final Exam - April 24th, 2008

Duration: 150 minutes

Name: ______________________ Student Number: ______________________

(1) **Do not open this test until instructed to do so!**
(2) **Please place your student ID (or another picture ID) on the desk.**
(3) This exam should have 4 pages, including this cover sheet.
(4) No textbooks, calculators, or other aids are allowed.
(5) Turn off any cell phones, pagers, etc. that could make noise during the exam.
(6) **Circle your solutions! Reduce your answer as much as possible. Explain your work.**

Read these UBC rules governing examinations:

(i) Each candidate must be prepared to produce, upon request, a Library/AMS card for identification.
(ii) Candidates are not permitted to ask questions of the invigilators, except in cases of supposed errors or ambiguities in examination questions.
(iii) No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.
(iv) Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action.
- Having at the place of writing any books, papers or memoranda, calculators, computers, audio or video cassette players or other memory aid devices, other than those authorized by the examiners.
- Speaking or communicating with other candidates.
- Purposely exposing written papers to the view of other candidates. The plea of accident or forgetfulness shall not be received.

(v) Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

1. [15pts]

(a) Find the directional derivative of $f(x, y, z) = e^{xyz}$ in the $(0, 1, 1)$ direction.
(b) Find the equation of the plane that contains $(1, 0, 0)$, $(0, 1, 0)$ and $(0, 0, 1)$.
(c) Using spherical coordinates and integration, show that the volume of the sphere of radius 1 centred at the origin is $4\pi/3$.
(d) Find $\nabla \left(y^2 + \sin(xy)\right)$.

2. [10pts] Calculate the integral:

$$\int_D \sin(y^2) dA$$

where D is the region bounded by $x + y = 0$, $2x - y = 0$, and $y = 4$.

3. [20pts] Consider the function $f(x, y, z) = x^2 + \cos(yz)$.

(a) Give the direction in which f is increasing the fastest at the point $(1, 0, \pi/2)$.
(b) Give an equation for the plane T tangent to the surface $S = \{f(x, y, z) = 1\}$ at the point $(1, 0, \pi/2)$.
(c) Find the distance between T and the point $(0, 1, 0)$.
(d) Find the angle between the plane T and the plane

$$P = \{x + z = 0\}.$$

4. [15pts] Consider the hemispherical shell bounded by the spherical surfaces

$$x^2 + y^2 + z^2 = 9 \quad \text{and} \quad x^2 + y^2 + z^2 = 4$$

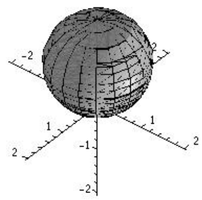
and above the plane $z = 0$. Let the shell have constant density D.

(a) Find the mass of the shell.
(b) Find the location of the center of mass of the shell.

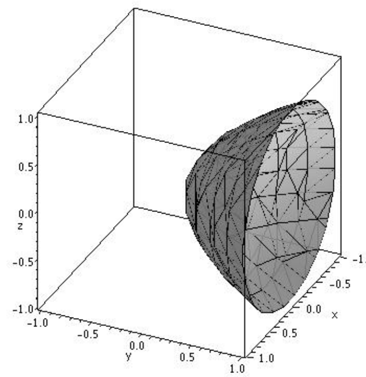
5. [10pts] Find the maximum and minimum values of $f(x, y) = x^2 + y^2$ subject to the constraint $x^4 + y^4 = 1$.

6. [10pts] Let E be the region bounded between the parabolic surfaces $z = x^2 + y^2$ and $z = 2 - x^2 - y^2$ and within the cylinder $x^2 + y^2 \leq 1$. Calculate the integral of $f(x, y, z) = (x^2 + y^2)^{3/2}$ over the region E.

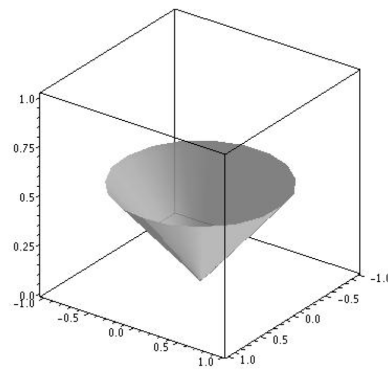
7. [10pts] Match the following equations and expressions with the corresponding pictures. Cartesian coordinates are (x, y, z), cylindrical coordinates are (r, θ, z), and spherical coordinates are (ρ, θ, ϕ).



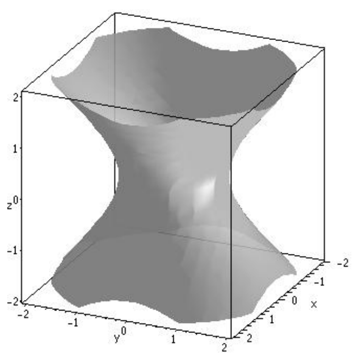
A



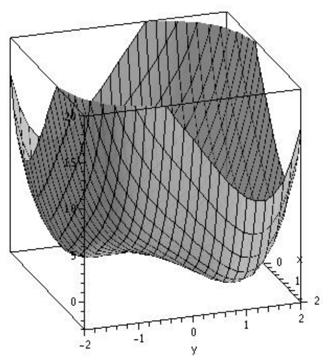
B



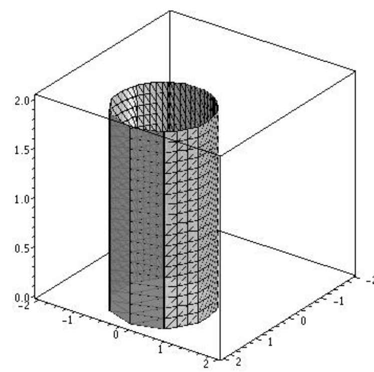
C



D



E



F

$\phi = \pi/3 \leftrightarrow$ ____________ $\qquad r = 2\cos\theta \leftrightarrow$ ____________

$x^2 + y^2 = z^2 + 1 \leftrightarrow$ ____________ $\qquad y = x^2 + z^2 \leftrightarrow$ ____________

$\rho = 2\cos\phi \leftrightarrow$ ____________ $\qquad z = x^4 + y^4 - 4xy \leftrightarrow$ ____________

8. [10pts] Write the integral given below 5 other ways, each with a different order of integration.

$$I = \int_0^1 \int_{\sqrt{x}}^1 \int_0^{1-y} f(x, y, z) dz dy dx.$$

You may find some of the following trig identities useful:

$$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$$
$$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$$
$$\sin(2a) = 2\sin(a)\cos(a) \quad \cos(2a) = 2\cos^2(a) - 1$$
$$\sin^2(a) = \frac{1-\cos(2a)}{2} \quad \cos^2(a) = \frac{1+\cos(2a)}{2}$$