

Name (print):

Student number:



University of British Columbia

MATH 110 (Section 003): DECEMBER EXAM

Date: *December 11, 2013*

Time: *12:00 noon to 2:30 p.m.*

Number of pages: *12 (including cover page)*

Exam type: *Closed book*

Aids: *No calculators or other electronic aids*

Rules governing formal examinations:

Each candidate must be prepared to produce, upon request, a UBC card for identification.

No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.

Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:

- *Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;*
- *Speaking or communicating with other candidates;*
- *Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.*

Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

For examiners' use only		
Question	Mark	Possible marks
1		8
2		5
3		3
4		5
5		6
6		5
7		4
8		4
9		5
10		5
Total		50

1. Determine whether each of the following statements is true or false. If it is true, provide justification. If it is false, provide a counterexample.

(a) [2 marks] The graph of $f(x) = \frac{1}{2}((x+3)^2 - 2)$ crosses the x -axis.

(b) [2 marks] If $\lim_{x \rightarrow 1} f(x) = 4$, then $f(1) = 4$.

(c) [2 marks] $f(x) = \begin{cases} \frac{3}{x+2} & \text{if } x < 1 \\ \sqrt{x} & \text{if } x \geq 1 \end{cases}$ is continuous at all real numbers.

(d) [2 marks] If $f(x)$ is differentiable at $x = 2$, then $\lim_{h \rightarrow 0} \frac{f(2+h) - f(2)}{h} = 2$.

2. [5 marks] Find the equations of the two tangent lines to the curve $y = \frac{x-4}{x+4}$ that are parallel to the line $x - 2y = 2$.

3. Let $f(x) = x^6$.

- (a) **[1 mark]** What is the derivative of $f(x)$?
- (b) **[2 marks]** Rewrite $f(x)$ in such a way that you can differentiate it using a different rule than the Power Rule. Name the rule of differentiation which is applicable, and then confirm that the derivative calculated using this alternative method is the same as in part (a).
- (c) **[3 bonus marks]** Can you do the same as in part (b), but with yet another rule? You will receive one bonus mark for each different differentiation rule you use to differentiate $f(x)$.

4. Let $f(x) = \sqrt{x} - 1$ and $g(x) = (f \circ f)(x)$.

- (a) [**1 mark**] Find a formula for $g(x)$ in terms of x (in other words, your formula should not use the symbol “ f ”).
- (b) [**2 marks**] What is the domain of $g(x)$?
- (c) [**2 marks**] Find the derivative of $g(x)$.

5. (a) **[3 marks]** Evaluate the following three quantities: $\sin\left(\frac{5\pi}{3}\right)$, $\cos\left(-\frac{\pi}{3}\right)$, $\tan\left(\frac{11\pi}{3}\right)$.
- (b) **[3 marks]** Find the equation of the line tangent to the curve $y = \frac{\cos x}{\sin x}$ at $x = \frac{5\pi}{3}$.

6. Consider the function:

$$f(x) = \frac{e^{(x^3)}e^{(2x^2)}e^x}{e^3}.$$

- (a) [**1 mark**] How many solutions are there to the equation $f(x) = 0$? Justify your answer.
- (b) [**2 marks**] Find the derivative of $f(x)$.
- (c) [**2 marks**] How many solutions are there to the equation $f'(x) = 0$? Justify your answer.

7. [4 marks] Find the derivative of the function

$$f(x) = (x^2 + 1)^7(x^4 + 2)^5(x^6 + 3)^3(x^8 + 4).$$

(There is no need to simplify your answer.)

8. [4 marks] Let

$$f(x) = \begin{cases} e^{-x} & \text{if } x < t \\ 2(x+1) & \text{if } x \geq t \end{cases}$$

Explain why there exists a number t such that f is continuous.

(Hint: apply the Intermediate Value Theorem to the function $e^{-t} - 2(t+1)$.)

9. [5 marks] Find the derivative of y with respect to x at the point $P = (0, \pi)$ along the curve:

$$\sin(x + y) = xy$$

10. (a) [**2 marks**] Explain what it means for a function to be differentiable at the point $x = 2$ in terms of limits.

(b) [**3 marks**] Find constants a and b so that the function

$$f(x) = \begin{cases} x^2 - 2x + 1 & \text{if } x \leq 2 \\ ax + b & \text{if } x > 2 \end{cases}$$

is differentiable everywhere.

This page may be used for rough work. It will not be marked.