

Name (print):

ID number:

Section (circle): 001 002 003



University of British Columbia
APRIL EXAM for MATH 110

Date: *April 24, 2012*

Time: *3:30 to 6:00 p.m.*

Number of pages: *13 (including cover page)*

Exam type: *Closed book*

Aids: *No calculators or other electronic aids*

Rules governing formal examinations:

Each candidate must be prepared to produce, upon request, a UBC card for identification.

No candidate shall be permitted to enter the examination room after the expiration of one-half hour from the scheduled starting time, or to leave during the first half hour of the examination.

Candidates suspected of any of the following, or similar, dishonest practices shall be immediately dismissed from the examination and shall be liable to disciplinary action:

- *Having at the place of writing any books, papers or memoranda, calculators, computers, sound or image players/recorders/transmitters (including telephones), or other memory aid devices, other than those authorized by the examiners;*
- *Speaking or communicating with other candidates;*
- *Purposely exposing written papers to the view of other candidates or imaging devices. The plea of accident or forgetfulness shall not be received.*

Candidates must not destroy or mutilate any examination material; must hand in all examination papers; and must not take any examination material from the examination room without permission of the invigilator.

Candidates must follow any additional examination rules or directions communicated by the instructor or invigilator.

For examiners' use only		
Question	Mark	Maximum mark
1		6
2		6
3		6
4		6
5		18
6		6
7		6
8		6
9		3 (bonus)
Total		60

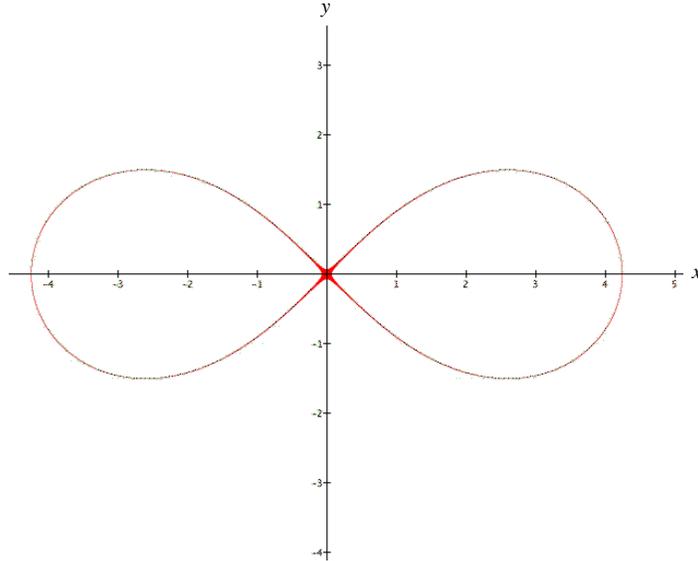
Your answers must be “calculator-ready”, but they do not have to be simplified.

This page may be used for rough work. It will not be marked.

1. [6 marks] The curve below is called a *lemniscate*; it is given by the equation

$$(x^2 + y^2)^2 = 18(x^2 - y^2).$$

Find the equation of the tangent line at the point $(2, \sqrt{2})$, and sketch the tangent line.



2. (a) [4 marks] Use a suitable linear approximation to estimate $\ln(1.1)$.

(b) [2 marks] Is your estimate in part (a) greater than, less than, or equal to the actual value of $\ln(1.1)$? Justify your answer.

3. (a) [**3 marks**] State either the Mean Value Theorem or Rolle's Theorem.

(b) [**3 marks**] Consider a function f whose derivative is equal to 0 on the entire interval $(-\infty, \infty)$. Let x_1 and x_2 be two points on $(-\infty, \infty)$. Prove, using your answer in part (a), that

$$f(x_2) - f(x_1) = 0.$$

4. [6 marks] Write down a piecewise algebraic expression for a function f satisfying all of the following criteria.

(a) f is defined on the interval $[-1, 1]$.

(b) f is discontinuous at $x = 0$ and $x = 1$, and continuous everywhere else on $[-1, 1]$.

(c) f has a global minimum at $x = 0$.

(d) f has no global or local maximum.

Make a large sketch of the graph of f as well.

5. Let $f(x) = \frac{2x^2}{x^2 - 1}$.

(a) [**1 mark**] State the domain of f .

(b) [**1 mark**] Find all the x - and y -intercepts of f .

(c) [**2 marks**] Find the horizontal asymptotes of f .

(d) [**2 marks**] Find the vertical asymptotes of f .

(e) [4 marks] Determine where f is increasing and where it is decreasing.

(f) [4 marks] Determine where f is concave up and where it is concave down.

- (g) [4 marks] Make a large sketch of the graph of f , indicating all you have found in the previous parts of the question.

6. In a fission nuclear weapon, a sphere of uranium-235 is compressed to a fraction of its size, forcing at least one atomic nucleus to split and release neutrons. These neutrons force other nuclei to release *their* neutrons, and so on. The result is that the number of neutrons grows exponentially.

Let $P(t)$ be the number of neutrons t seconds after detonation. The number of neutrons satisfies the differential equation

$$\frac{dP}{dt} = 10^8 P(t).$$

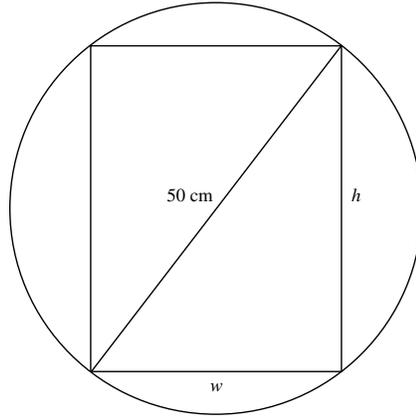
- (a) [**2 marks**] Write down an exponential algebraic expression for $P(t)$ which satisfies the differential equation given above.

- (b) [**4 marks**] According to your model in part (a), how long does it take a given number of neutrons to double?

7. [6 marks] The strength of a wooden beam whose cross section is of width w and height h is given by the formula

$$S = kwh^2,$$

where k is a positive constant. Consider a 50 cm-diameter log, pictured below. What is the width and height of the strongest beam that can be cut from this log?



8. [6 marks] Oceanographers on research ships use a device called a *CTD probe* to make underwater measurements at extreme depths. The probe drops vertically through the water at 4 m/s. Simultaneously, the research ship moves away from the drop site at 5 m/s. What is the rate of change of the distance between the probe and the ship 10 seconds after the probe is dropped?

9. [**3 bonus marks**] An *antiderivative* of a function f is a function F satisfying $F'(x) = f(x)$ for all x . Find two antiderivatives of the function e^{3-x} .