This summer I worked with Anstee on forbidden Berge hypergraphs. Forbidden Berge hypergraphs are similar to forbidden configurations and patterns. We explored this problem using matrices with 0,1 entries. Define a matrix $A$ to be simple if it is a $(0,1)$ matrix with no repeated columns. We can interpret the rows as vertices of a hypergraph and a column $j$ as a hyperedge with a 1 in row $i$ if vertex $i$ is included in hyperedge $j$. Let $F$ be a $(0,1)$ $k \times l$ matrix. We say that $F$ is a Berge hypergraph of $A$ and write $F \prec A$ if there is a $k \times l$ submatrix of $A$ that has a row and column permutation, call it $G$, such that $G \geq F$. Let

$$BAvoid(m, F) = \{A : A \text{ is } m\text{-rowed, simple, } F \not\prec A \text{ for all } F \in F\},$$

$$Bh(m, F) = \max_{A} \{\|A\| : A \in BAvoid(m, F)\}.$$ 

$Bh(m, F)$ is the extremal function we are interested in. In forbidden configurations we write $F \prec A$ and say $F$ is a configuration of $A$ if there exists a row and column permutation a submatrix of $A$, say $G$ with $F = G$. With forbidden configurations we have

$$Avoid(m, F) = \{A : A \text{ is } m\text{-rowed, simple, } F \not\prec A \text{ for all } F \in F\},$$

$$\text{forb}(m, F) = \max_{A} \{\|A\| : A \in Avoid(m, F)\}.$$ 

In most cases we consider $F = \{F\}$ in which case we write $Bh(m, F) = Bh(m, F)$. We establish a complete classification of all $Bh(m, F)$ where $F$ is a $k \times l$ matrix with $k \leq 5$, with the exception of one missing maximal result for $k = 5$. We also provide

*Research supported in part by NSERC USRA
many general results and draw an interesting connection to a problem explored by Alon and Shikhelman[1].

**Theorem 0.1** Let $I_k = K^1_k$ denote the $k \times k$ identity matrix. Then

$$\text{Bh}(m, I_k) = 2^{k-1}.$$  

The classification for the $\Theta(1)$ and $\Theta(m)$ boundary follows from this result and a result in forbidden configurations. It is worth noting that $\text{forb}(m, I_k)$, the extremal function of interest in forbidden configurations, is significantly different.

**Lemma 0.2** Given $A \in \text{BAvoid}(m, F)$, there exists a matrix $T(A) \in \text{BAvoid}(m, F)$ with $\|A\| = \|T(A)\|$ and $T_i(T(A)) = T(A)$ for $i = 1, 2, \ldots, m$.

Let $S$ be the set system of $A$; this lemma implies that $S$ is a downset, i.e if $S \subseteq S$ and $S' \subseteq S$ then $S' \subseteq S$. A result of this lemma is that $\text{Bh}(m, K_2 \times K_t) = \Theta(\text{ex}(m, K_3, K_2 \times K_t))$, where $\text{ex}(m, K_3, K_2 \times K_t)$ is the number of triangles in a graph avoiding the complete bipartite graph on 2 and $t$ vertices. This specific problem, as well as the general problem of $\text{ex}(m, G, F)$, the maximum number of subgraphs $G$ in a graph avoiding $F$ as a subgraph, has been addressed by Alon and Shikhelman in [1]. From our work this summer we believe the problems of interest to be $\text{Bh}(m, F)$ where $F = I_{a_1} \times I_{a_2} \times \cdots \times I_{a_p}$, a generalization of the problem posed by Zarankiewicz[2].

We also obtained a classification of growth rates $\text{forb}(m, F)$ where $F$ is the vertex-edge incidence matrix of a forest. Let

$$H_1 = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}.$$  

**Theorem 0.3** Assume $k \geq 5$ and let $F$ be the $k \times l$ vertex-edge incidence matrix of a forest $T$.

1. $\text{forb}(m, F)$ is $\Theta(m^{k-3})$ if and only if $F \prec H_1$

2. $\text{forb}(m, F)$ is $\Theta(m^{k-2})$ if and only if $F \nprec H_1$ and $T$ has at most 2 vertices of degree $\geq 3$ and those two vertices are connected by a path of at most 2 or not connected.

3. if $F$ is not one of the two previous cases, then $\text{forb}(m, F)$ is $\Theta(m^{k-1})$.

**References**
