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Symmetry Methods for Differential Equations
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This summer I worked with Dr. George Bluman on updating his 2010 book "Applications of Symmetry Methods to Partial Differential Equations", written with Alexei Cheviakov and Stephen Anco.

A differential equation (DE) describes a family of solution surfaces, and a symmetry of a DE is a mapping from solution surfaces to other solution surfaces (or themselves). Every non-trivial DE admits symmetries; the goal of symmetry methods is to find symmetries of a given DE and use them effectively in solving or simplifying the DE. Symmetry methods generalize the collection of disparate techniques for solving DE's that are taught in most undergraduate DE courses, and can be implemented systematically. For these reasons, symmetry methods are used in modern DE solving software. An example of an application of symmetry methods is the determination of mappings between PDE's, that is, mappings that relate the solutions of a given PDE to the solutions of a different PDE. Many more techniques are known for solving linear PDEs with constant coefficients than linear PDEs with variable coefficients. By analyzing the symmetries of a given linear PDE with variable coefficients, one can determine whether there exists an invertible mapping of the given linear PDE to a linear PDE with constant coefficients, and construct the mapping if it exists. Similarly, through symmetry analysis one can determine whether a nonlinear PDE can be invertibly mapped to a linear PDE.

For the project, I first did background reading to become acquainted with the subject matter. I then read through "Applications of Symmetry Methods to Partial Differential Equations", noting errors and suggesting where the writing could be made clearer. The new edition of the book will include new developments in the field, so I assisted in a literature search through papers that have cited Dr. Bluman's work. I recorded bibliographic information of papers that appeared to be relevant, and then looked at those papers more carefully to determine which ones were of interest. In some cases I performed calculations to verify that a paper's results were indeed novel. For example, sometimes a symmetry is claimed to be dependent on higher order derivatives, but a substitution can be made to eliminate the higher order derivative terms.