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Families of Forbidden Configurations

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In this research we consider Families of forbidden configurations. The study of forbidden configurations is considering matrices the do not contain a *configuration* and trying to maximize the size of such matrices.

We call a (0,1) matrix is *simple* if it contains no repeated columns. We say a simple (0,1)-matrix A *contains* a (0,1)-matrix F as a configuration if there is a sub-matrix of A that is a row and column permutation of F. We denote the number of columns of A by $||A||$.
In the study of forbidden configurations we are interested in calculating the extremal function $Forb(m, F)$; corresponding to the maximum number of columns an m-rowed simple matrix A can have such that A does not contain F as a configuration.

One can consider *families of forbidden configurations*, corresponding to forbidding multiple such F as mentioned above, denoting these F as the family being forbidden. In previous years, these have been explored, and an operator + has been introduced. This operator is meant to be used to be able to combine different families of forbidden configurations into one. Among different families, some were more crucial in better understanding the behaviour of this operator. Namely $\mathcal{F}_{2,p}$ and $\mathcal{F}_{3,p}$.

These two families are defined for any $p > 0$ as

$$\mathcal{F}_{2,p} = \{p \begin{bmatrix} 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \cdot p \begin{bmatrix} 1 \\ 1 \end{bmatrix}\}$$

$$\mathcal{F}_{3,p} = \{p \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \cdot \begin{bmatrix} 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 0 & 1 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 & 0 & 1 \end{bmatrix} \cdot p \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}\}$$

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It was previously established that

$$pm/2 \leq Forb(m, \mathcal{F}_{2,p}) \leq 2pm$$

We were able to tighten the bound this year to

$$pm/2 \leq Forb(m, \mathcal{F}_{2,p}) \leq pm$$

This was done by considering the implication of not containing $\mathcal{F}_{2,p}$ on each pair of rows: not containing $\mathcal{F}_{2,p}$ limits what these two rows could look like. We construct a graph, with vertices corresponding to rows, and edges to pair of rows and what component of $\mathcal{F}_{2,p}$ is missing on the pair. By considering the properties of this graph we were able to tighten bound.

In the case of $\mathcal{F}_{3,p}$, we hoped to achieve $Forb(m, \mathcal{F}_{3,p}) \leq \binom{m}{2} + O(m)$. Some results were found and documented[1] in this effort, though they were not enough to prove this result.

[1] All obsevations and results are documented in *Forbidden Configurations Observations*, a document by F. Fallahi under Supervision of R.P. Anstee