

# Dynamics of Granular Toys

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NSERC USRA Report

Summer 2011

## 1 Abstract / Introduction

Piles of granular materials display fluid-like behaviour when they avalanche under gravity, yet behave like solids once they come to rest. This project explores phenomena related to rocking and rolling granular drums. Cylinders either empty or completely full of grains rolls like solid objects down an incline. However, if the cylinder is partially full, the dynamics are far more complicated, becoming unsteady and erratic as grains avalanche within the rolling drum.

It has been known since Coulomb's time that above a threshold angle  $\phi_0$ , granular materials begin to avalanche until the threshold angle is achieved. This is related to its macroscopic static friction coefficient  $\gamma_s$  by

$$\tan \phi_0 = \gamma_s.$$

From Newtonian mechanics, we know that for an object to be "stuck" to another by friction, the following equation must hold:

$$F \leq \mu_s N$$

where  $\mu_s$  is the static coefficient of friction between the bottom contact line of the granular bed and the cylinders inner surface. Thus, when we roll a partially-full cylinder down an incline, there are four possible states the model can be in: when the top layer of the grains are avalanching or non-avalanching, and if the bottom layer of the grains is sticking or slipping with respect to the inner surface of the cylinder.

After deriving a system of 5 ODEs for all four states, several numerical simulations were produced based on various cases. Finally, physical experiments were done to confirm the numerical behaviour of the model.

## 2 Conclusion

In the case where it is easier for the grains to slip than avalanche ( $\mu_s < \gamma_s$ ), as we increase the angle of incline  $\alpha$ , the model goes from a fixed equilibrium, to a non-avalanching "wobble" about a fixed point, to rolling down the incline without any avalanching, to rolling down the incline with an avalanching limit cycle, and finally to rolling down the incline with a continuously increasing granular flow. Strange behaviour involving a delayed avalanching limit cycle occurs when the model changes from periodic to ballistic motion and the angle of the incline  $\alpha$  is approximately  $7.26^\circ$  (here,  $\mu_s = 0.4$  and  $\gamma_s = 0.6$ ).

In the case where it is easier for the grains to avalanche than slip ( $\mu_s > \gamma_s$ ), as we increase the angle of incline  $\alpha$ , the model goes from a fixed equilibrium, to a non-avalanching wobble about a fixed point, to rolling down the incline with a continuously increasing granular flow while the wedge of sand is stuck to the cylinder, to finally rolling down the incline with a continuously increasing granular flow and the wedge of sand unstuck from the cylinder. The model switches from periodic to ballistic motion at approximately  $\alpha = 7.18^\circ$  when  $\mu_s = 0.6$  and  $\gamma_s = 0.4$ .

For all diagrams,  $x(\text{m})$ ,  $v(\text{m/s})$ ,  $\phi(\text{rad})$ ,  $d\phi/dt(\text{rad/s})$  and  $q(\text{m}^2/\text{s})$ . Initial conditions are so that the granular bed starts at the bottom and that the cylinder is initially at rest, i.e.  $\mathbf{x}_0 = (\frac{\pi}{2}, 0, 0, 0, 0)$ .

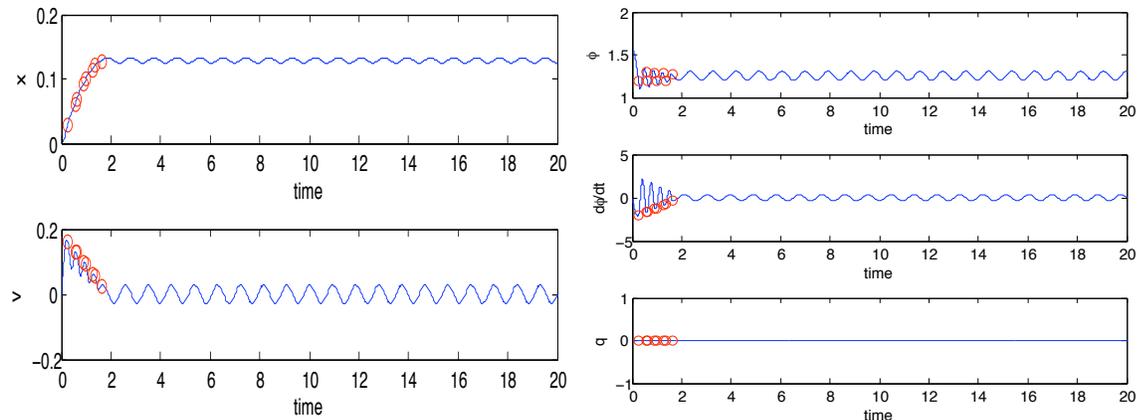


Figure 1: The “wobbling” state, with  $\alpha = 6.5^\circ$ ,  $\mu_s = 0.6$  and  $\gamma_s = 0.4$

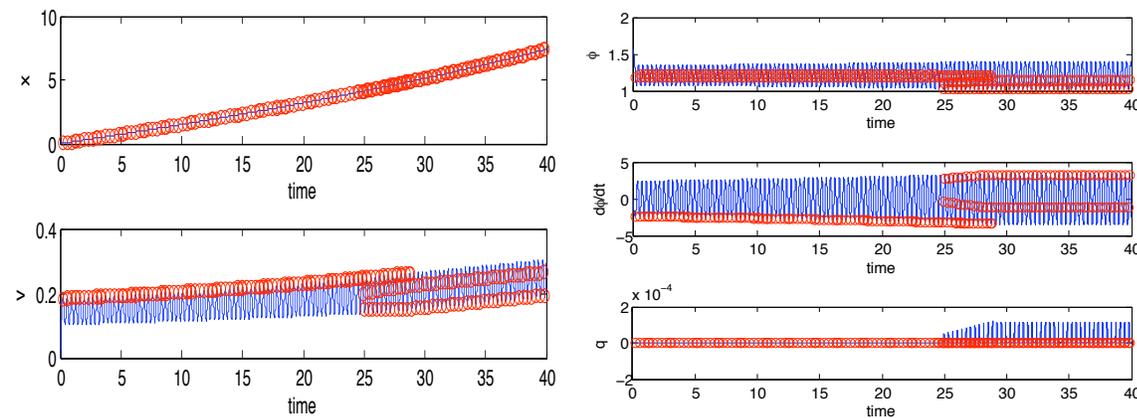


Figure 2: The delayed avalanching limit cycle state, with  $\alpha = 7.27^\circ$ ,  $\mu_s = 0.6$  and  $\gamma_s = 0.4$ .

### 3 References

Fauve, S. et al. “Dynamics of Avalanches in a Rotating Cylinder.” *Mobile Particulate Systems*. Ed. E. Guazzelli and L. Oger. Springer, 1995. 331-366. Print.

Special thanks to Dr. Neil J. Balmforth for his supervision on this project.