Equality of Schur $Q$-functions

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In learning about quasisymmetric functions I was naturally led to investigate equality of Schur $Q$-functions. What follows is three of my most significant results produced during my summer research, which will result in a journal article joint with Steph van Willigenburg.

1 Definitions

• Let $\alpha^\circ$ and $\alpha^t$ represent, respectively, the rotation by 180 degrees and transpose of the composition $\alpha$.

• Let $r_\alpha$ denote the skew Schur $Q$-function whose shifted diagram corresponds to the ribbon $\alpha$.

• Let $s_D$ denote the skew Schur $Q$-function whose shifted diagram corresponds to the ordinary skew diagram $D$.

• Let $\bullet$ represent the bullet operation: $\alpha\bullet\beta$ means that we take $|\alpha|$ copies of $\beta$, alternatively transpose them, and then glue them according to $\alpha$.

2 Three of my results

**Theorem 2.1** For compositions $\alpha$, $\beta$ and skew diagram $D$, if $r_\alpha = r_\beta$ then $s_\alpha\bullet D = s_\beta\bullet D$.

**Theorem 2.2** The skew diagram $\alpha_1 \bullet \cdots \bullet \alpha_m \bullet D$ has the same ordinary skew Schur $Q$-function as those skew diagrams $\beta_1 \bullet \cdots \bullet \beta_m \bullet E$ where $\beta_i = \{\alpha_i, \alpha_i^t, \alpha_i^\circ, (\alpha_i^t)^\circ\}$ for $1 \leq i \leq m$, and $E = \{D, D^t, D^\circ, (D^t)^\circ\}$.

**Theorem 2.3** (i) For $|\alpha|$ odd, the ribbon Schur $Q$-function $r_\alpha$ is irreducible considered as an element of $\mathbb{Z}[q_1, q_3, \ldots]$.

(ii) For $|\alpha|$ even, there are infinitely many examples in which $r_\alpha$ is irreducible and infinitely many examples in which $r_\alpha$ is reducible.

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