This summer I worked with Dr. Richard Anstee on the problems of forbidden submatrices and configurations, topics of extremal combinatorics. Given a $k \times l$ matrix $F$ whose entries are all 0's or 1's (a $(0,1)$-matrix), we consider an $m$-rowed $(0,1)$-matrix $A$ with no repeated columns ($A$ is simple), and no submatrix $F$. We define $\text{Avoids}(m, F)$ to be the set of such matrices $A$, and $fs(m, F)$ to be the maximum number of columns of any $A \in \text{Avoids}(m, F)$. There is a conjecture of Anstee, Frankl, Füredi, and Pach that $fs(m, F) \in O(m^k)$. Similarly, we can consider the problem of forbidding any row or column permutation of $F$ (a configuration of $F$), defining $\text{Avoid}(m, F)$ to be the set of simple $(0,1)$-matrices $A$ with no configuration $F$ and $\text{forb}(m, F)$ to be the maximum number of columns of such an $A$. We seek to prove bounds on $fs(m, F)$ and $\text{forb}(m, F)$ for specific $F$ to gain insight.

1 Forbidden Submatrices

A structural result was achieved for the submatrix

$$F = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}. $$

Keeping track of instances of $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and noting that if it occurs on distant rows an instance is present on every intermediate row, we motivate the following definition.

**Definition 1.1** The span $C_\alpha$ of a column $\alpha$ is the set of rows between its top 1 and bottom 0, inclusive.

For example, if $\alpha = (0, 1, 1, 0, 1)^T$, $C_\alpha = \{2, 3, 4\}$. The following is our result.
Lemma 1.2 There exists a matrix $A \in Avoids(m, \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix})$ with $|C_\alpha|$ increasing.

Additionally, we made some observations regarding the submatrix

$$ F = \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}. $$

From constructions avoiding the submatrix

$$ \Gamma = \begin{bmatrix} 1 & 1 \\ 1 & 0 \end{bmatrix} $$

we considered columns of a given column sum and made the following definitions:

Definition 1.3 A primary column is a column whose top 1 is in a row where no previous column of that column sum had its top 1.

Definition 1.4 A secondary column is a column that is not a primary column.

Note that every secondary column creates the submatrix $\Gamma$ with a primary column. There are $m - k + 1$ primary columns of $B_k$ given by the choices of the locations of the top 1, so we aim to produce a bound on the number of secondary columns. We can assign to every secondary column $\beta$ a row $j$ such that $\Gamma$ occurs with right column $\beta$ and bottom row $j$. If row $j$ is already associated with a row $\delta$, we could show that a new row $k$ could be assigned to a column to resolve this conflict. However, it is possible that a column $\beta$ would be assigned a row $j$, creating an overlap that assigns it to row $k$, and conflicting with a previous column to assign it back to row $j$. The presence of these cycles prevented us from proving a linear bound. A number of results regarding these cycles were provided, however.

2 Forbidden Configurations

We attempted to produce a quadratic bound for the configuration

$$ F = t \cdot \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{bmatrix}. $$

Using inductive techniques applied to a previous configuration, along with a result of Balogh and Bollobás, we attempted to deduce the structure of $A$. 
References
