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Eigenvalues and eigenfunctions on random graphs

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This summer I had the pleasure to work with Professor Omer Angel and Professor Lior Silberman to investigate eigenfunction on random graphs. I was also working with two other students, Shamil Asgarli and Yuqi Zhu, in this project. The two classes of random graphs we studied were random 4 regular graphs and random quadrangulations. We started our investigation by simulating the random graphs using Matlab and Python programming languages. Specifically, the simulations we had can construct the adjacency matrices for the random graphs, and find the eigenvalues and the eigenfunctions.

In our investigation of random 4 regular graphs, we used the random permutation model to construct the adjacency matrix and implemented inverse iteration to find the eigenfunction corresponds to the eigenvalue closest to the chosen value. The main goal for this investigation was showing the eigenfunction distribution is close to normal. We tested normality using Kolmogorov-Smirnov test and third and fourth moments. The experimental results showed the distribution was close to normal, but analytical result was not yet obtained.

To help us constructing random quadrangulations in the simulation we used Schaeffer's Bijection, which says there is a bijection between well-labelled trees and quadrangulations. We first looked at the spectrum of the random walk matrix and analysed the features appeared in the spectrum. The spectrum showed some spikes at several eigenvalues; specifically, the spike at eigenvalue 0 showed the nullity of the matrix is close to  $1/4$ . We found the spikes in the spectrum came from the local features in the random quadrangulations. That is, some local features in the random quadrangulations show up with high frequencies and increase the multiplicities of the corresponding eigenvalues. We achieved to show the eigenvalues correspond to local features are dense in the set  $(-1, 1)$ . We also came up with the constructions of the local features that could produce any eigenvalues that is square rational in the set  $(-1, 1)$ . In the investigation for the eigenvalue 0 in the random quadrangulations, we found for any eigenvalue 0, the corresponding eigenfunction must take 0 at all vertices which are neighbours of the leaves in the quadrangulation. Base on this observation, we found the nullity of a random quadrangulation is equal to the nullity of the graph after removing the leaves and

the neighbours of the leaves, plus the number of leaves and minus the number of the neighbours of the leaves in the quadrangulation. We further analysed the multiplicity  $1/4$  by calculating the probability for a vertex in UIPQ to be a leaf or a neighbour of a leaf. We studied Schaeffer's Bijection to see what local features in the well-labelled trees produce leaves and neighbours of leaves in quadrangulations. The recursion formula we had could be used to analyse the probabilities of the frequency of any local features in the well-labelled trees, and the analytical results do agree with experimental results.