

KPZ LINE ENSEMBLE: A MARRIAGE OF INTEGRABILITY AND PROBABILITY

ALAN HAMMOND

The KPZ equation, introduced by Kardar, Parisi and Zhang, is a stochastic PDE that models randomly evolving interfaces that are subject to constraining forces such as surface tension. It is anticipated to be a universal object, in the sense that many microscopic models will share the KPZ equation as an accurate asymptotic description of their late time behavior. This view is supported by extensive numerical evidence, recent experimental evidence involving liquid crystal instabilities, and a limited but growing body of mathematically rigorous work.

In recent work arXiv:1312.2600 with Ivan Corwin, we present a new technique for the analysis of the KPZ equation. The solution to the equation is represented as the lowest indexed curve in an N -indexed ensemble of curves, which we call a KPZ line ensemble. Curves within the ensemble enjoy a natural invariance under resampling, the H-Brownian Gibbs property, which property has the effect of energetically penalizing, but not absolutely forbidding, the crossing of adjacently indexed curves. This property is inherited from the O'Connell-Yor semi-discrete continuum random polymer ensemble after a limiting procedure is applied.

The H-Brownian Gibbs property is an integrable one, in the sense that the precursor O'Connell-Yor ensemble is known to enjoy it by virtue of this ensemble's algebraic structure. However, it also offers a powerful probabilistic tool for the analysis of the KPZ equation. Since the solution of this equation is embedded in a KPZ line ensemble, we may analyse it using the H-Brownian Gibbs property, and in this way derive significant new estimates regarding the regularity and local structure of the KPZ solution. As I will aim to explain, these new estimates are valid uniformly in the time parameter for the KPZ equation, even after a natural rescaling of the equation is undertaken which accesses the fluctuation behavior of the KPZ evolution.