Every problem is worth 10 points.

**Problem 1:** Let $\sum_{n=1}^{\infty} a_n$ be a convergent series of positive terms. Show that the series $\sum_{n=1}^{\infty} \frac{a_n}{n+1}$ also converges.

Hint: consider the set $S := \{ n : a_n/\sqrt{n} \leq 1/2 \}$.

**Problem 2:** Find all entire functions $f(x + iy) = u(x) + iv(y)$, with the real valued functions $u(x)$ and $v(y)$ depending only on $x$ and $y$ respectively.

**Problem 3:** Consider the vector field $F = \langle xy^2 + z, x^2 y + 2, x \rangle$. Evaluate the line integral $\int_C F \cdot dr$ where $C = (3t/\pi, \sin t, \cos t)$ for $t \in [0, \pi]$.

**Problem 4:** Evaluate $I = \int_{-\infty}^{\infty} \frac{\cos x}{(x^2 + 1)(x^2 + 4)} \, dx$.

**Problem 5:** Let $f(x)$ be a uniform limit of real differentiable functions $f_n(x)$ on $[-1, 1]$. Assume that $|f'_n(x)| \leq C$ for some $C$ independent of $n$ and $x \in [-1, 1]$. Recall that, under these assumptions, $f(x)$ is always continuous.

(a) Is the function $f(x)$ necessarily differentiable? If so, prove it. If not, provide a counterexample.

(b) Suppose that, in addition, the derivatives $f'_n(x)$ converge pointwise to $g(x)$ and $f(x)$ is differentiable. Then, is it necessarily true that $f'(x) = g(x)$? If so, prove it. If not, provide a counterexample.

**Problem 6:** Find the maximum value of $|(1 - z)e^z|$ in the region $|z| \leq 1$. 