1. (10 points) (a) Determine if the following series converges or diverges:

\[ \sum (-1)^n \frac{\sqrt{n}}{1 + \sqrt{n}} \]

(b) For which values of \( \alpha > 0 \) does the following series converge?

\[ \sum \frac{1}{n^\alpha (\log n)^2} \]

(c) Find the radius of convergence of the following power series:

\[ \sum \frac{z^n}{(1 + (-1)^n)^2 + (1 - (-1)^n)3^n} \]

2. (10 points) Let \( S \) be the part of the cylinder \((x + y + 1)^2 + z^2 = 4\) which lies in the first octant. Find the flux of \( \vec{F} \) upwards through \( S \) where

\[ \vec{F} = xy \hat{i} + (z - xy) \hat{j} \]

3. (10 points) Let \( I \) be a bounded interval in \( \mathbb{R} \) and \( f_n \) be continuous functions on \( I \) such that \( f_{n+1}(x) \leq f_n(x) \) for all \( x \in I, n \in \mathbb{N} \). Suppose that \( f_n(x) \) converges to 0 for each \( x \in I \).

(a) Give a counterexample to show that the conditions above do not imply that \( f \to 0 \) uniformly.

(b) Suppose that \( I \) is compact. Prove that \( f \to 0 \) uniformly.

4. (10 points) How many zeros does the polynomial \( z^4 + \frac{1}{4}z^3 - \frac{1}{4} \) have in the annulus \( \{ z \in \mathbb{C} : \frac{1}{2} < |z| < 1 \} \)?

5. (10 points) Determine for which integer values of \( n \) (positive, negative, or 0), there exists a holomorphic function defined in the region \( |z| > 1 \), whose derivative is

\[ \frac{z^n}{1 + z^2} \]

6. (10 points) Let \( D = \{ z \in \mathbb{C} : |z| < 1 \} \), and suppose that \( f : D \to \mathbb{C} \) is holomorphic, and injective when restricted to \( D \setminus \{0\} \). Prove that \( f \) is injective.