

MATH 444: Writing Mathematically

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This is just *my* rambling thoughts on writing or indeed speaking Mathematically. One goal for all Mathematics students is that they acquire this skill seeking both clarity and precision. Some students acquire this quite easily, others find it quite hard and need lots of practice. It is akin to learning how to write an essay with slightly different rules and objectives. Reading Mathematics is one way to learn.

Notation is often present in Mathematical discourse. Think of the variable x . When we talk about a *set* we have certain mathematical expectations while when we use the word set in a general setting it may lose much of its Mathematical meaning. For example a set of dishes used at a party. It is stylistically nice when the notation is obviously a Mathematical notation. Think eigenvalue. Or Function. We immediately think of their Mathematical definitions (which you may have to give in your writeup to make it more self-contained). But you can check some news articles to find Mathematical words used in non mathematical contexts.

Your writing/speaking should strive to be clear. No extraneous details. This can cause trouble in a number of ways. You may need to define some terms. The words increasing, positive come to mind. Does it make sense to say an increasing function has a positive derivative? A *strictly positive number* is an obvious object but if you have already defined positive this way you don't have to repeat it every time. You would almost never worry about defining the *real numbers* \mathbf{R} in a discussion despite the fact that this is significant Mathematics. You may, however, appeal to properties of the real numbers such as completeness.

If you are describing a proof then the broad outline of the proof technique should be made clear. For example:

This proof will use induction on m .

This proof will be by contradiction.

This proof splits into 3 cases of which two are relatively easy.

This proof will be done by first proving a series of Lemmas.

Statements of results should clearly state the hypothesis and the conclusions. In particular, if certain things are fixed, then it should be made clear when they are fixed. An example is a theorem that says given any constant c , then ... In logic terms, this is the notion of scope.

In the real world, definitions can be hard to come by. Can you really define the deficit or indeed debt of the Province of British Columbia? There are quite clear accounting rules that are followed but they are complicated rules and many people don't understand them. They distinguish at one point, from an accounting perspective, is a revenue or expense to be recorded. And there are different types of debt. While this may seem somewhat theoretical, it is actually these notions of debt which keeps UBC from borrowing on the open market to build additional residences for students.

Motivation for a result is important. A simple but demonstrative example of a Theorem is desirable.

There are plenty of ideas for good writing that should be taken from an English composition course. But occasionally the ideas don't translate so easily to writing about Mathematical results. The notion of important keywords used as notation are big in Mathematics. Different examples will be treated equally. Different cases will be treated equally. By this I mean that the reader should see that the different examples are from the same theorem. Or the different cases all have to be proved to obtain the main result. In an english essay you might be encouraged to use a variety of expressions.