Eta Matrices

In computer implementations of the Revised Simplex Method it is sometimes useful to give the updates to the matrix B in a concise way. In particular, the *eta* matrix E is the matrix that updates B:

$$(\text{new } B) = (\text{old } B)E$$

We explained in class, that this update can be added to an LU factorization of old$B$ and we write new$B = LUE$ and are still able to quickly solve for $\mathbf{y}$ in $\mathbf{t}^T = \mathbf{c}^T_{\text{new}B}\text{new}B^{-1}$ and for $\mathbf{a}$ in new$BA_k = \mathbf{a}$. We note that the new B after a pivot differs in only one column from the old B in only one column and so the matrix E has a very simple form. Imagine that the jth column of B as been replaced while variable x_k enters and x_l leaves. Thus E is the identity matrix where the jth column has been replaced by $(\text{old } B)^{-1}A_k$ where A_k is the column of $[AI]$ indexed by x_k. Computer implementations of the Revised Simplex Method manage to avoid computing B^{-1} but do use the eta matrices. Read Chapter 7 in the text for more details if you wish.

Let me give some examples from the sample of the revised simple method that is posted on the web. There is one strange anomaly below that may confuse you; namely one of the matrices is its own inverse! This happens from time to time.

$$B_3^{-1} = \begin{bmatrix} x_4 & x_5 & x_3 \\ 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix}^{-1} = \begin{bmatrix} x_4 & x_5 & x_3 \\ 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{bmatrix} = B_3.$$

$$B_1 = \begin{array}{c} \\ x_4 \\ x_5 \\ x_6 \end{array} \begin{array}{c} \begin{array}{ccc} x_4 & x_5 & x_6 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{array}$$

x_0 enters, x_6 leaves

$$B_2 = \begin{array}{c} \\ x_4 \\ x_5 \\ x_6 \end{array} \begin{array}{c} \begin{array}{ccc} x_4 & x_5 & x_0 \end{array} \\ \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{pmatrix} \end{array} = B_1E_1 = \begin{array}{c} \\ x_4 \\ x_5 \\ x_6 \end{array} \begin{array}{c} \begin{array}{ccc} x_4 & x_5 & x_6 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \end{array} \begin{array}{c} \\ x_4 \\ x_5 \\ x_6 \end{array} \begin{array}{c} \begin{array}{ccc} x_4 & x_5 & x_0 \end{array} \\ \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{pmatrix} \end{array}$$

x_3 enters, x_0 leaves

$$B_3 = \begin{array}{c} \\ x_4 \\ x_5 \\ x_6 \end{array} \begin{array}{c} \begin{array}{ccc} x_4 & x_5 & x_3 \end{array} \\ \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{pmatrix} \end{array} = B_2E_2 = \begin{array}{c} \\ x_4 \\ x_5 \\ x_6 \end{array} \begin{array}{c} \begin{array}{ccc} x_4 & x_5 & x_0 \end{array} \\ \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -1 \\ 0 & 0 & -1 \end{pmatrix} \end{array} \begin{array}{c} \\ x_4 \\ x_5 \\ x_0 \end{array} \begin{array}{c} \begin{array}{ccc} x_4 & x_5 & x_3 \end{array} \\ \begin{pmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix} \end{array}$$

x_1 enters, x_5 leaves

$$B_4 = \begin{array}{c} \\ x_4 \\ x_5 \\ x_6 \end{array} \begin{array}{c} \begin{array}{ccc} x_4 & x_1 & x_3 \end{array} \\ \begin{pmatrix} 1 & -4 & 2 \\ 0 & -1 & 2 \\ 0 & 1 & -1 \end{pmatrix} \end{array} = B_3E_3 = \begin{array}{c} \\ x_4 \\ x_5 \\ x_6 \end{array} \begin{array}{c} \begin{array}{ccc} x_4 & x_5 & x_3 \end{array} \\ \begin{pmatrix} 1 & 0 & 2 \\ 0 & 1 & 2 \\ 0 & 0 & -1 \end{pmatrix} \end{array} \begin{array}{c} \\ x_4 \\ x_5 \\ x_3 \end{array} \begin{array}{c} \begin{array}{ccc} x_4 & x_1 & x_3 \end{array} \\ \begin{pmatrix} 1 & -2 & 0 \\ 0 & 1 & 0 \\ 0 & -1 & 1 \end{pmatrix} \end{array}$$

x_2 enters, x_4 leaves

$$B_5 = \begin{array}{c} \\ x_4 \\ x_5 \\ x_6 \end{array}\!\!\begin{array}{c} \begin{array}{ccc} x_2 & x_1 & x_3 \end{array} \\ \begin{pmatrix} 1 & -4 & 2 \\ -3 & -1 & 2 \\ 1 & 1 & -1 \end{pmatrix} \end{array} = B_4 E_4 = \begin{array}{c} \\ x_4 \\ x_5 \\ x_6 \end{array}\!\!\begin{array}{c} \begin{array}{ccc} x_4 & x_1 & x_3 \end{array} \\ \begin{pmatrix} 1 & -4 & 2 \\ 0 & -1 & 2 \\ 0 & 1 & -1 \end{pmatrix} \end{array} \begin{array}{c} \\ x_4 \\ x_1 \\ x_3 \end{array}\!\!\begin{array}{c} \begin{array}{ccc} x_2 & x_1 & x_3 \end{array} \\ \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -2 & 0 & 1 \end{pmatrix} \end{array}$$