Example of Cycling (from Chvatal)

This example has the virtue of suffering from no roundoff errors when run on a computer. Cycling in LP's remains rare and so many implementations do not implement an anti-cycling rule. We use Anstee's pivot rules (which are otherwise known as the standard rules) to pivot into the basis the variable with the largest coefficient in the z row (and in the case of ties take the variable of smallest index) and for the leaving variable we break ties, if necessary, by choosing the variable of smallest index. We typically expect you to follow these pivot rules on test questions.

$$\text{dictionary 1}\quad \begin{array}{rclrrrr} x_5 & = & & -0.5x_1 & +5.5x_2 & +2.5x_3 & -9x_4 \\ x_6 & = & & -0.5x_1 & +1.5x_2 & +0.5x_3 & -x_4 \\ x_7 & = & 1 & -x_1 & & & \\ z & = & & 10x_1 & -57x_2 & -9x_3 & -24x_4 \end{array}$$

We have x_1 enter and x_5 leave.

$$\text{dictionary 2}\quad \begin{array}{rclrrrr} x_1 & = & & -2x_5 & +11x_2 & +5x_3 & -18x_4 \\ x_6 & = & & x_5 & -4x_2 & -2x_3 & +8x_4 \\ x_7 & = & 1 & +2x_5 & -11x_2 & -5x_3 & +18x_4 \\ z & = & & -20x_5 & +53x_2 & +41x_3 & -204x_4 \end{array}$$

We have x_2 enter and x_6 leave.

$$\text{dictionary 3}\quad \begin{array}{rclrrrr} x_1 & = & & +.75x_5 & -2.75x_6 & -.5x_3 & +4x_4 \\ x_2 & = & & .25x_5 & -.25x_6 & -.5x_3 & +2x_4 \\ x_7 & = & 1 & -.75x_5 & -13.25x_6 & +.5x_3 & -4x_4 \\ z & = & & -6.75x_5 & -13.25x_6 & +14.5x_3 & -98x_4 \end{array}$$

We have x_3 enter and x_1 leave.

$$\text{dictionary 4}\quad \begin{array}{rclrrrr} x_3 & = & & +.1.5x_5 & -5.5x_6 & -2x_1 & +8x_4 \\ x_2 & = & & -.5x_5 & +2.5x_6 & +x_1 & -2x_4 \\ x_7 & = & 1 & & & -x_1 & \\ z & = & & 15x_5 & -93x_6 & -29x_1 & +18x_4 \end{array}$$

We have x_4 enter and x_2 leave.

$$\text{dictionary 5}\quad \begin{array}{ccccccc} x_3 & = & & -.5x_5 & +4.5x_6 & +2x_1 & -4x_2 \\ x_4 & = & & -.25x_5 & +1.25x_6 & +.5x_1 & -.5x_2 \\ x_7 & = & 1 & & & -x_1 & \\ z & = & & 10.5x_5 & -70.5x_6 & -20x_1 & -9x_2 \end{array}$$

We have x_5 enter and x_3 leave.

$$\text{dictionary 6}\quad \begin{array}{ccccccc} x_5 & = & & -2x_3 & +9x_6 & +4x_1 & -8x_2 \\ x_4 & = & & +.5x_3 & -x_6 & -.5x_1 & +1.5x_2 \\ x_7 & = & 1 & & & -x_1 & \\ z & = & & -21x_3 & +24x_6 & +22x_1 & -93x_2 \end{array}$$

We have x_6 enter and x_4 leave.

$$\text{dictionary 7}\quad \begin{array}{ccccccc} x_5 & = & & -0.5x_1 & +5.5x_2 & +2.5x_3 & -9x_4 \\ x_6 & = & & -0.5x_1 & +1.5x_2 & +0.5x_3 & -x_4 \\ x_7 & = & 1 & -x_1 & & & \\ z & = & & 10x_1 & -57x_2 & -9x_3 & -24x_4 \end{array}$$

Thus we have returned to dictionary 1 (which is not surprising since we have returned to the same basis $\{x_5, x_6, x_7\}$). We call this cycling since we would repeat this over and over ad infinitum following Anstee's rule. Bland's Rule avoids cycling. In dictionary 6 there are two choices for entering variables and so we choose x_1 to enter and then to have x_4 leave.

$$\text{dictionary 8}\quad \begin{array}{ccccccc} x_5 & = & & +2x_3 & +x_6 & -8x_4 & +4x_2 \\ x_1 & = & & +.x_3 & -2x_6 & -2x_4 & +3x_2 \\ x_7 & = & 1 & -x_3 & +2x_6 & +2x_4 & -3x_2 \\ z & = & & x_3 & -20x_6 & -44x_4 & -27x_2 \end{array}$$

We have x_3 enter and x_7 leave (a non degenerate pivot!).

$$\text{dictionary 9}\quad \begin{array}{ccccccc} x_5 & = & 2 & -2x_7 & +5x_6 & -4x_4 & -2x_2 \\ x_1 & = & 1 & -.x_7 & & & \\ x_3 & = & 1 & -x_7 & +2x_6 & +2x_4 & -3x_2 \\ z & = & 1 & -x_7 & -18x_6 & -42x_4 & -30x_2 \end{array}$$

Thus we have reached optimality.