One challenge when doing Mathematics is to choose the right level of generality. In the context of Linear Algebra there are at least three levels of generality. We began the course with 2×2 matrices

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

and a number of results were proven by considering the 4 entries individually. We then proved a number of results using matrices. Finally we also prove results by considering linear transformations, a special class of functions which can be associated with a matrix. Thus we have three levels of generality:

1. entries of the matrix e.g. a, b, c, d
2. matrices e.g. A, B, A^{-1}
3. linear transformations e.g. f, $f \circ g$, f^{-1}

Our first proof of the product rule for determinants for 2×2 matrices $(\det(AB) = \det(A)\det(B))$ considered the entries individually. We generalized to $n \times n$ matrices by obtaining a matrix proof that any invertible matrix is a product of elementary matrices and the product rule works for elementary matrices.

Our proof of the associative rule for matrix multiplication was done using linear transformations using the easy fact that function composition is associative. Our proof that A^{-1} is both a left and right inverse is best done using the idea that for a function f, the compositional inverse f^{-1} is unique.

When faced with a new problem it is not clear which level of generality to use; the good news is that you have three possibilities to find the proof. Often there are many ways to get to the answer. Typically, if you can get the proof to work, it is preferable to use the higher level of generality.

But there are more challenges than just this. We can view a matrix in many ways. It can be viewed as a linear transformation. If the matrix is invertible it can be viewed as a change of coordinate matrix. We might view the matrix as a set of column vectors. Or we might view it as a set of row vectors. And at least for one lecture the matrix encodes a block design. This would be only a small sampling of the many possible interpretations of a matrix.

Thus when faced with a problem in Linear Algebra, it might be quite difficult to chose the 'right' level of generality or the 'right' matrix interpretation.