# Forbidden Configurations: Asymptotic bounds

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# Introduction

I have worked with a number of coauthors in this area: Farzin Barekat, Laura Dunwoody, Ron Ferguson, Balin Fleming, Zoltan Füredi, Jerry Griggs, Nima Kamoosi, Steven Karp, Peter Keevash, Linyuan Lu, Christina Koch, Connor Meehan, U.S.R. Murty, Miguel Raggi and Attila Sali but there are works of other authors impinging on this problem as well.

Survey at www.math.ubc.ca/∼anstee

Jerry isn't tall


One birdwatcher and one pack animal

Jerry in South Carolina


Deynise, Malia and Jerry in South Carolina

# Simple Matrices and Set Systems

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i.e. if $A$ is $m$-rowed then $A$ is the incidence matrix of some family $\mathcal{A}$ of subsets of $[m] = \{1, 2, \ldots, m\}$.

$$A = \begin{bmatrix} 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

$$\mathcal{A} = \{\emptyset, \{2\}, \{3\}, \{1,3\}, \{1,2,3\}\}$$

# Configurations

**Definition** Given a matrix $F$, we say that $A$ has $F$ as a *configuration* written $F \prec A$ if there is a submatrix of $A$ which is a row and column permutation of $F$.

$$F = \begin{bmatrix} 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{bmatrix} \quad \prec \quad \begin{bmatrix} 1 & 1 & 1 & 1 & 1 & 1 \\ 0 & \boxed{1 \quad 0 \quad 1} & & 1 & \boxed{0} \\ 0 & 0 & 0 & 0 & 1 & 1 \\ 0 & \boxed{1 \quad 1 \quad 0} & & 0 & \boxed{0} \end{bmatrix} = A$$

# Our Extremal Problem

**Definition** We define $\|A\|$ to be the number of columns in $A$.

$\text{Avoid}(m, \mathcal{F}) = \{\, A \,:\, A \text{ is } m\text{-rowed simple, } F \nprec A \text{ for } F \in \mathcal{F}\}$

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There are other possibilities for extremal problems for Avoid$(m, F)$ including maximizing the weighted sum over columns where a column of column sum $i$ is weighted by $1/\binom{m}{i}$ (e.g. Johnson and Lu) or maximizing the number of 1's .

# A Product Construction

As with any extremal problem, the results are often motivated by constructions, namely matrices in Avoid$(m, F)$. Early investigations with Jerry Griggs and Attila Sali suggested a product construction might be very helpful.

The building blocks of our product constructions are $I$, $I^c$ and $T$:

$$I_4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad I_4^c = \begin{bmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{bmatrix}, \quad T_4 = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

**Definition** Given an $m_1 \times n_1$ matrix $A$ and a $m_2 \times n_2$ matrix $B$ we define the product $A \times B$ as the $(m_1 + m_2) \times (n_1 n_2)$ matrix consisting of all $n_1 n_2$ possible columns formed from placing a column of $A$ on top of a column of $B$. If $A$, $B$ are simple, then $A \times B$ is simple. (A, Griggs, Sali 97)

$$\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix} \times \begin{bmatrix}1&1&1\\0&1&1\\0&0&1\end{bmatrix} = \left[\begin{array}{ccc|ccc|ccc}1&1&1&0&0&0&0&0&0\\0&0&0&1&1&1&0&0&0\\0&0&0&0&0&0&1&1&1\\\hline 1&1&1&1&1&1&1&1&1\\0&1&1&0&1&1&0&1&1\\0&0&1&0&0&1&0&0&1\end{array}\right]$$

Given $p$ simple matrices $A_1, A_2, \ldots, A_p$, each of size $m/p \times m/p$, the $p$-fold product $A_1 \times A_2 \times \cdots \times A_p$ is a simple matrix of size $m \times (m^p/p^p)$ i.e. $\Theta(m^p)$ columns.

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# The Conjecture

**Definition** Let $x(F)$ denote the largest $p$ such that there is a $p$-fold product which does not contain $F$ as a configuration where the $p$-fold product is $A_1 \times A_2 \times \cdots \times A_p$ where each $A_i \in \{I_{m/p}, I^c_{m/p}, T_{m/p}\}$.

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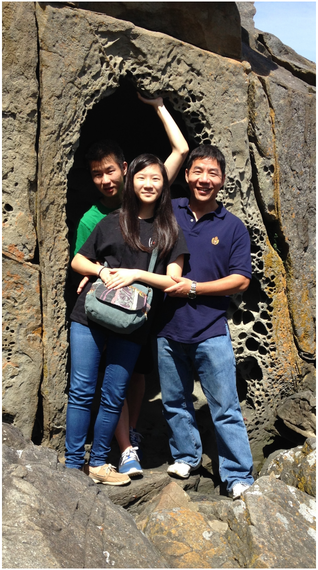
Attila Sali



Linyuan and his kids

# An Unavoidable Forbidden Family

**Theorem** (Balogh and Bollobás 05) Let $k$ be given. Then

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**Theorem** (A., Lu 14) Let $k$ be given. Then there is a constant $c$

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If you take all columns of column sum at most $k-1$ that arise from the $k-1$-fold product $T_{k-1} \times T_{k-1} \times \cdots \times T_{k-1}$ then this yields $\binom{2k-2}{k-1} \approx 2^{2k}$ columns. A probabalistic construction in Avoid$(m, \{I_k, I_k^c, T_k\})$ has $2^{ck \log k}$ columns.

# Ramsey Theory

Our proof uses lots of induction and multicoloured Ramsey numbers: $R(k_1, k_2, \ldots, k_\ell)$ is the smallest value of $n$ such than any colouring of the edges of $K_n$ with $\ell$ colours $1, 2, \ldots, \ell$ will have some colour $i$ and a clique of $k_i$ vertices with all edges of colour $i$. These numbers are readily bounded by multinomial coefficients:

$$R(k_1, k_2, \ldots, k_\ell) \leq \binom{\sum_{i=1}^{\ell} k_i}{k_1\ k_2\ k_3 \cdots\ k_\ell}$$

$$R(k_1, k_2, \ldots, k_\ell) \leq \ell^{k_1 + k_2 + \cdots + k_\ell}$$

Our first proof had something like $forb(m\{, I_k, I_k^c, T_k\}) < R(R(k,k), R(k,k))$ yielding a doubly exponential bound.

Let $u = R(k, k+1, k, k, k+1, k) < 6^{6k+2}$.

As part of our proof we show that for $A \in \text{Avoid}(m, \{I_k, I_k^c, T_k\})$, $A$ cannot have a $u \times 2u$ (0,1)-submatrix of the form

| | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 0 | 1 | * | * | * | * | * | * | * | * | * | * |
| $a$ | $a$ | 0 | 1 | * | * | * | * | * | * | * | * |
| $b$ | $b$ | $c$ | $c$ | 0 | 1 | * | * | * | * | * | * |
| $d$ | $d$ | $e$ | $e$ | $f$ | $f$ | 0 | 1 | * | * | * | * |
| $g$ | $g$ | $h$ | $h$ | $i$ | $i$ | $j$ | $j$ | 0 | 1 | * | * |
| $k$ | $k$ | $l$ | $l$ | $m$ | $m$ | $n$ | $n$ | $o$ | $o$ | 0 | 1 |

One can interpret the entries of the matrix as $1 \times 2$ blocks yielding a $u \times u$ matrix with the blocks below the diagonal either $\boxed{0\ 0}$ or $\boxed{1\ 1}$ with blocks on the diagonal $\boxed{0\ 1}$ and arbitrary (0,1)-blocks above the diagonal.

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There are 6 colours to consider

$$\begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & * \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} * & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & * \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} * & 0 \\ 1 & 1 \end{bmatrix}$$

We are able to show that $u < R(k, k+1, k, k, k+1, k)$ and so we get a singly exponential bound on $u \leq 6^{6k+2}$. The proof has more to do than this but this is a critical step.

We say that the edge $i, j$ is colour $\begin{bmatrix} 1 & * \\ 0 & 0 \end{bmatrix}$ if we have $0\,0$ in entry $(j, i)$ and $1\,*$ in entry $(i, j)$:

$$\begin{array}{ccc} & i & & j \\ i & 0\,1 & & 1\,* \\ & & \ddots & \\ j & 0\,0 & & 0\,1 \end{array}$$

Now consider a clique of size $k+1$ of colour $\begin{bmatrix} 1 & * \\ 0 & 0 \end{bmatrix}$:

$$\begin{array}{cccccccccc} 0 & 1 & 1 & * & 1 & * & 1 & * & 1 & * \\ 0 & 0 & 0 & 1 & 1 & * & 1 & * & 1 & * \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & * & 1 & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array}$$

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$$\begin{array}{cc|c|c|c|c|c|c|cc} 0 & 1 & 1 & * & 1 & * & 1 & * & 1 & * \\ 0 & 0 & 0 & 1 & 1 & * & 1 & * & 1 & * \\ 0 & 0 & 0 & 0 & 0 & 1 & 1 & * & 1 & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & * \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \quad \text{yields } T_k$$

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Thus if we have $u = R(k, k+1, k, k, k+1, k)$ then we find a copy of $I_k$ or $I_k^c$ or $T_k^c$. a contradiction to $A \in \text{Avoid}(m, \{I_k, I_k^c, T_k\})$. We conclude $u < R(k, k+1, k, k, k+1, k)$ and so we get a singly exponential bound $u \leq 6^{6k+2} = 2^{ck}$.

# Multiple Columns

Let $s \cdot F$ denote $\overbrace{[F|F|\cdots|F]}^{s}$.

**Theorem** (A, Füredi 86) Let $s \geq 2$ be given. Let $K_k$ denote the $k \times 2^k$ simple matrix of all possible (0,1)-columns on $k$ rows. Then $forb(m, s \cdot K_k)$ is $\Theta(m^k)$.

**Theorem** (A, Lu 14) Let $s \geq 2$ be given. Then there exists a (largish) constant $C$ with

$$forb(m, \{s \cdot I_k, s \cdot I_k^c, s \cdot T_k\}) \leq (4s-3)m + C$$

.

# Exact bounds and asymptotic bounds

$$\text{Let} \quad F = \begin{bmatrix} 1 & 0 \\ 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{bmatrix}$$

**Theorem** (Frankl, Füredi, Pach 87) $forb(m, F) = \binom{m}{2} + 2m - 1$ i.e. $forb(m, F)$ is $\Theta(m^2)$.

**Theorem** (A. and Lu 13) Let $s$ be given. Then $forb(m, s \cdot F)$ is $\Theta(m^2)$.

Note $x(F) = 2 = x(s \cdot F)$ so this is consistent with conjecture

**Problem** Let $\alpha$ be given and imagine $m^\alpha$ as an honourary integer. Show $forb(m, m^\alpha \cdot F)$ is $\Theta(m^{2+\alpha})$.

We can only prove that $forb(m, m^\alpha \cdot F)$ is $O(m^{\min\{3+\alpha, 2+2\alpha\}})$.

We say a matrix with entries in $\{0, 1, \ldots, r-1\}$ is an *r*-matrix.

An *r*-matrix is simple if there are no repeated columns.

$forb(m, r, \mathcal{F}) = \max\{\|A\| \,:\, A \text{ is a simple } r\text{-matrix}, F \not\prec A, F \in \mathcal{F}\}$

We say a matrix with entries in $\{0, 1, \ldots, r-1\}$ is an $r$-matrix.

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$forb(m, r, \mathcal{F}) = \max\{\|A\| : A$ is a simple $r$-matrix, $F \nprec A, F \in \mathcal{F}\}$

**Theorem** (A, Lu 14) Given $r$ there exists a constant $c_r$ so that $forb(m, \mathcal{T}_k(r)) \leq 2^{c_r k^2}$.

$\mathcal{T}_k(3)\backslash\mathcal{T}_k(2) =$

$$\begin{bmatrix} 1 & 2 & \cdots & 2 \\ 2 & 1 & \cdots & 2 \\ \vdots & \vdots & \ddots & \\ 2 & 2 & \cdots & 1 \end{bmatrix}, \begin{bmatrix} 0 & 2 & \cdots & 2 \\ 2 & 0 & \cdots & 2 \\ \vdots & \vdots & \ddots & \\ 2 & 2 & \cdots & 0 \end{bmatrix}, \begin{bmatrix} 2 & 0 & \cdots & 0 \\ 0 & 2 & \cdots & 0 \\ \vdots & \vdots & \ddots & \\ 0 & 0 & \cdots & 2 \end{bmatrix},$$

$$\begin{bmatrix} 2 & 1 & \cdots & 1 \\ 1 & 2 & \cdots & 1 \\ \vdots & \vdots & \ddots & \\ 1 & 1 & \cdots & 2 \end{bmatrix}, \begin{bmatrix} 2 & 2 & \cdots & 2 \\ 0 & 2 & \cdots & 2 \\ \vdots & \vdots & \ddots & \\ 0 & 0 & \cdots & 2 \end{bmatrix}, \begin{bmatrix} 2 & 2 & \cdots & 2 \\ 1 & 2 & \cdots & 2 \\ \vdots & \vdots & \ddots & \\ 1 & 1 & \cdots & 2 \end{bmatrix},$$

$$\begin{bmatrix} 0 & 0 & \cdots & 0 \\ 2 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \\ 2 & 2 & \cdots & 0 \end{bmatrix}, \begin{bmatrix} 1 & 1 & \cdots & 1 \\ 2 & 1 & \cdots & 1 \\ \vdots & \vdots & \ddots & \\ 2 & 2 & \cdots & 1 \end{bmatrix}.$$

**Problem** Let $\mathcal{F}$ be a family of $(0,1)$-matrices.
$forb(m, (\mathcal{T}_k(3)\backslash\mathcal{T}_k(2) \cup \mathcal{F}))$ is $\Theta(forb(m, \mathcal{F}))$.

THANKS to Linyuan and all those who helped in organizing this wonderful event.

And of course Happy Birthday to Jerry!!