Math 267, Section 202 : HW 4

All **five** questions are due **Wednesday, January 30th**.

1. Consider the periodic signal, period $\mathtt{T} = 4$, given by:

$$g(x) = \begin{cases} 0 & \text{for } -2 < x < -1 \\ x & \text{for } -1 < x < 1 \\ 0 & \text{for } 1 < x < 2 \end{cases}$$

   (and repeated periodically.)

   Compute the (complex) Fourier series of $g(x)$.

2. Let $f(x)$ be a 2-periodic function (i.e. period $T = 2$) and

$$f(x) = \begin{cases} 0 & \text{for } 100 < x < 101 \\ x & \text{for } 101 \leq x < 102 \end{cases}$$

   (a) Determine the value of $f(301.5)$.

   (b) Determine the (complex) Fourier series of $f(x)$.

3. Consider two 2π-periodic signals $f(t)$ and $g(t)$. Suppose we only know that the Fourier coefficient c_k of $f(t)$ and the Fourier coefficient d_k of $g(t)$ satisfy

$$\begin{cases} c_k = d_k & \text{for } -100 \leq k \leq 100 \text{ ,} \\ c_k = d_k + 3^{-|k|} & \text{for } |k| > 100. \end{cases}$$

   Compute $\int_{-\pi}^{\pi} |f(t) - g(t)|^2 dt$.

4. Consider the function, defined for $0 < x < 2$ by:

$$f(x) = \begin{cases} 0 & \text{for } 0 < x < 1 \\ 1 & \text{for } 1 < x < 2 \end{cases}$$

   (a) Sketch the graph of *even extension*, $f_{even}(x)$.

   (b) Compute the (complex) Fourier series of the *even extension*.

   (c) What is the sum of the resulting Fourier series for $x = 2$? Give a numeric value.

5. For $-1 < x < 1$,

$$x^3 = \sum_{k \neq 0} i \left(\frac{1}{k\pi} - \frac{3}{2} \frac{1}{k^3 \pi^3} \right) e^{i\,k\,\pi\,x}.$$

   Rewrite this as a *real Fourier series*.