Math 267, Section 202 : HW 9

Due **Wednesday, March 27th**.

1. [Convolution of non-periodic signals]

   Recall for integers $n \in \mathbb{Z}$,

   $$u[n] = \begin{cases} 1 & \text{if } n \geq 0 \ , \\ 0 & \text{otherwise.} \end{cases} \qquad \delta[n] = \begin{cases} 1 & \text{if } n = 0, \\ 0 & \text{otherwise.} \end{cases} \qquad \delta_{n_0}[n] = \begin{cases} 1 & \text{if } n = n_0, \\ 0 & \text{otherwise.} \end{cases}$$

   Recall the class example $(u * u)[n] = (n+1)u[n]$.

   (a) Find $(\overbrace{\delta_2 * \delta_2 * \cdots * \delta_2}^{100 \text{ times}})[n]$.

   (b) Let $f[n] = u[n-2]$. $g[n] = u[n+3]$.

      i. Find $(f * u)[n]$.

      ii. Find $(f * g)[n]$.

   (c) Let

   $$h[n] = \begin{cases} 1 & |n| \leq 3 \ , \\ 0 & \text{otherwise.} \end{cases}$$

   Find $(h * u)[n]$

      i. first, by computing the convolution sum directly;

      ii. second, by using the algebraic properties of the convolution and using $(u * u)[n] = (n+1)u[n]$.

2. [Discrete-time Fourier transform for non-periodic signals]

   (a) $x[n] = \delta_2[n] + \delta_{-2}[n]$

   (b) $y[n] = \left(\frac{1}{5}\right)^n u[n-1]$

   (c) $z[n] = \left(\frac{1}{5}\right)^{|n+1|}$

3. **[NOT TO HAND IN]** [Inverse discrete-time Fourier transform for non-periodic signals]

   Recall the discrete-time Fourier transforms of $\delta_{n_0}[n]$ and $a^n u[n]$ (for $|a| < 1$) are $e^{-i\omega n_0}$ and $\frac{1}{1-ae^{-i\omega}}$, respectively.

   Use these to find discrete-time signals $x[n]$, $y[n]$, $z[n]$, whose Fourier transforms are given below. (Here, each answer should be a signal defined on the set of integers: $n \in \mathbb{Z}$.)

   (a) $\widehat{x}(\omega) = \cos^2 \omega + \cos \omega \sin \omega$. (Hint: Can we express this as combination of complex exponentials?)

(b)

$$\widehat{y}(\omega) = 1 + \frac{e^{i2\omega}}{1 + \frac{1}{3}e^{-i\omega}}$$

(Hint; you may want to use time-shift property: see the table in page 12 in the online note " Discrete-time Fourier series and Fourier Transforms".)

(c)

$$\widehat{z}(\omega) = \frac{1}{(1 + \frac{1}{2}e^{-i\omega})(1 + \frac{1}{3}e^{-i\omega})}.$$

(Hint: use partial fractions.)

4. Let $x[n] = \left(\frac{1}{3}\right)^n u[n]$ and $y[n] = \left(\frac{1}{5}\right)^n u[n]$

   (a) Find $(x * y)[n]$ by directly computing the convolution summation.

   (b) Find $(x * y)[n]$ by applying DTFT.

   (c) Check whether you get the same answer from (a) and (b).

5. Use DTFT to find a discrete time signal $y[n]$ that satisfies

$$y[n] - \frac{1}{4}y[n-2] = \delta[n-2] \quad \text{for all integer } n.$$

   (Hint: you may need to do partial fractions.)

6. Consider an LTI system given by the following difference equation:

$$y[n] - 3y[n-1] = x[n] \qquad \text{for all integers } n.$$

   (a) Find the impulse response function $h[n]$ satisfying $h[n] = 0$ for all $n < 0$. For this case, find $y[n]$ when $x[n] = u[n]$.

   (b) Find the impulse response function $h[n]$ satisfying $h[0] = 0$. For this case, find $y[n]$ when $x[n] = u[n]$.