Math 267, Section 202 : HW 3

There are FIVE problems in two pages.

All questions are due **Monday January 28st**. Staple your HW paper!

1. Let

$$f(x) = 3\sin(3\pi x) + 5\sin(7\pi x), \qquad g(x) = \sum_{k=1}^{100} \sin(k\pi x/3),$$

$$h(x) = \sum_{k=1}^{50} \sin(k\pi x/3), \qquad p(x) = \sum_{k=100}^{200} k^2 \sin(k\pi x/3).$$

   Calculate the following:

   (a) $\int_0^3 f(x)^2 dx$

   (b) $\int_0^3 f(x)p(x)dx$

   (c) $\int_0^3 g(x)h(x)dx$

   (d) $\int_0^3 h(x)p(x)dx$

   (e) $\int_0^3 g(x)p(x)dx$

2. (a) Find all nontrivial (i.e. not identically zero) solutions of type

$$u(x,t) = X(x)T(t),$$

   to the following heat equation and given boundary conditions

$$\begin{cases} u_t = 9u_{xx} \ , & 0 < x < 3, \, t > 0, \\ u(0,t) = 0 = u(3,t), & t > 0. \end{cases}$$

   Here, you have to solve a relevant eigenvalue problem.

   (b) Use (a) to find a general solution solving the same equation and the boundary conditions:

   (c) Use (a) and (b) to find the solution to the same equation and the boundary conditions, and moreover the following initial condition:

$$u(x,0) = 1.$$

3. Solve the following initial-boundary value problem of the heat equation:

$$\begin{cases} u_t = 4u_{xx} \ , & 0 < x < 2, \, t > 0, \\ u(0,t) = 0 = u(2,t), & t > 0 \\ u(x,0) = f(x), & 0 < x < 2. \end{cases}$$

   where the function $f(x)$ is given by

$$f(x) = \begin{cases} 0 & \text{for } 0 < x < 1 \ , \\ x - 1 & \text{for } 1 \le x < 2. \end{cases}$$

4. Consider the following initial-boundary value problem of the heat equation:

$$\begin{cases} u_t = 4u_{xx} \ , & & 0 < x < 2, \, t > 0, \\ u(0,t) = 1 \quad \& & u(2,t) = 9, & t > 0, \\ u(x,0) = g(x), & & 0 < x < 2. \end{cases}$$

(a) Find a steady state.

(b) Solve the above initial-boundary value problem for

$$g(x) = \begin{cases} -1 & \text{for } 0 < x \le 1, \\ 1 & \text{for } 1 < x < 2. \end{cases}$$

5. Consider the following wave equation with boundary conditions:

$$\begin{cases} u_{tt} = 4u_{xx}\ , & & 0 < x < 2,\ t > 0, \\ u_x(0,t) = 1 \quad \& & u(2,t) = -3, & t > 0, \end{cases}$$

(Notice the derivative in one of the boundary conditions.)

(a) Find a steady state.

(b) Find a *general* solution that satisfies the wave equation and the boundary conditions above. (Notice that we do not have initial conditions).