Math 267, Section 202 : HW 10

Questions will **not** be collected. Solutions will be posted shortly.

1. Use the definition to calculate the z-transform and region of convergence of each signal. Sketch the region of conference, and mark the poles.
   (a) $x[n] = \left(\frac{1}{5}\right)^n u[n] + 2^n u[-n-1]$
   (b) $y[n] = (3-4i)^n\, u[n]$
   (c) $a[n] = \left(\frac{1}{3}\right)^{|n|}$
   (d) $b[n] = 3^{n+2} u[n-8]$
2. In each case, invert the z-transform to find $x[n]$
   (a) $X(z) = \frac{z^2}{z^2+z+1-i}$ and ROC $|z| < 1$
   (b) $X(z) = \frac{z}{(z-3)(z^2-i\,z+\frac{3}{4})}$ and ROC $\frac{3}{2} < |z| < 3$
3. Calculate the following convolutions, *using the z-transform.*
   (a) $(x * y)[n]$ when $x[n] = 2^n u[n]$ and $y[n] = \frac{1}{4}^n u[n+3]$
   (b) $(x * y)[n]$ when $x[n] = u[1-n]$ and $y[n] = (-1)^n u[-n]$

   Can you do the same thing using DTFT?
4. Suppose an LTI system
$$y[n] - 3y[n-1] = x[n]$$
   is *causal*.
   (a) Find $y[n]$ for $x[n] = 5^n u[n]$.
   (b) Find $y[n]$ for $x[n] = -5^n u[-n]$.
5. Consider the LTI given by the difference equation:
$$6\,y[n+2] - y[n+1] - y[n] = x[n]$$
   (a) Find the system function $H(z)$. (The system function is by definition, the z-transform of the impulse response function $h[n]$.)
   (b) Plot the poles of $H(z)$, and state the *possible* regions of convergence.
   (c) Find the impulse response $h[n]$ that is *neither left or right-sided*. (Here, $h[n]$ is called left-sided if $h[n] = 0$ for all large positive number n, i.e. $n \geq n_0$ for a positive number n_0, and it is called right-side if $h[n] = 0$ for all sufficiently negative numbers n, i.e. $n < n_0$ for a negative number n_0.

(d) When the impulse response function $h[n]$ is given from part(c), calculate the output for the input $x[n] = u[n+4] - u[n]$.

6. Consider the LTI given by the difference equation:

$$y[n+1] - 2y[n] + y[n-1] = i\, y[n-1] + x[n+1]$$

(a) Find the system function $H(z)$. (The system function is by definition, the z-transform of the impulse response function $h[n]$.)

(b) Plot the poles of $H(z)$, and state the *possible* regions of convergence.

(c) Find the impulse response $h[n]$ that makes the LTI *causal*.

(d) Calculate the output when $x[n] = \delta_{-1}[n] + \delta_2[n]$